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# Sage code for "A nonabelian conjecture of Birch and Swinnerton-Dyer type
# for hyperbolic curves", by J. Balakrishnan, I. Dan-Cohen, M. Kim,
# S. Wewers"
```

```
# We check that  $\text{Li}_2(\zeta_6) \neq 0$  for all primes  $p \equiv 1 \pmod{3}$ ,  $p \leq N$ ,
# see loc.cit., p. 18f
```

```
#N=100000
```

```
print "Checking that  $\text{Li}_2(\zeta_6) \neq 0$  for all  $p \equiv 1 \pmod{3}$ ,  $p \leq$ ",N
print
```

```
check=True
badprimes=set()
```

```
for p in prime_range(N):
```

```
    if p.mod(3)==1:
```

```
        k=FiniteField(p)
        R.<v>=k[]
        #print "p=",p
```

```
        g0=v-v^p
        g1=-R(g0/(v-v^2)).integral()
        g2=-R(g1/(v-v^2)).integral()
        #print "g_2=",g2
```

```
        a=k.primitive_element()
        b=a^((p-1)/6)
        #print "zeta_6=",b
```

```
        if g2(b)==0:
            print "for p=",p,", zeta=",b,"is a root of g_2"
            check=False
            badprimes.add(p)
```

```
if check:
```

```
    print "Check was successful for all  $p \leq$ ",N
```

```
else:
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```
    print "Check unsuccessful; bad primes are:",badprimes
```