Homework 1

1. i^n is:

n=4k	n=4k+1	n=4k+2	n=4k+3
1	i	-1	-i

2. i. $\frac{x}{x^2+y^2}$. ii. $\frac{-2ixy}{(x^2+y^2)^2}$. iii. x^2-y^2+x. iv. 2xy.
3. 5+2i, 1-6i, 5-12i, 5+i, -88-16i, $\frac{-1-8i}{10}$, $\frac{-68+24i}{13}$, $\frac{-6+8i}{5}$
4. i. $2^{52}(i-1)$
   ii. $\frac{1}{\sqrt{13}}^{47}[cos(-47\alpha)+isin(-47\alpha)],\ \alpha = arctan(\frac{3}{2})$
   iii. $\frac{1}{65}^{76}[cos(-152\alpha)+isin(-152\alpha)],\ \alpha = arctan(\frac{1}{8})$
   iv. $\pm(\sqrt{1+\frac{\sqrt{5}}{2}}-\frac{\sqrt{1+\frac{\sqrt{5}}{2}}}{2+\sqrt{5}}i)$;$\pm(\sqrt{\frac{c+\sqrt{c^2+1}}{2}}+\ \sqrt{\frac{1}{2}(c-\sqrt{c^2+1}}\ i)$
   v. 1
5. i. 0.75+i. ii. x=2, y=-3. iii. $x=y=\pm\frac{1}{\sqrt{2}}$
6. proof
7. $2(cos(\pi)+isin(\pi));\ 2(cos(\frac{1}{2}\pi)+isin(\frac{1}{2}\pi));2(cos(\frac{3}{2}\pi)+isin(\frac{3}{2}\pi));$
   $\sqrt{2}(cos(\frac{1}{4}\pi)+isin(\frac{1}{4}\pi));\sqrt{2}(cos(\frac{7}{4}\pi)+isin(\frac{7}{4}\pi));$
   $2(cos(\frac{2}{3}\pi)+isin(\frac{2}{3}\pi));2(cos(\frac{11}{6}\pi)+isin(\frac{11}{6}\pi));(cos(\frac{1}{6}\pi)+isin(\frac{1}{6}\pi))$
8. $\{4^{\frac{1}{4}}cos(\frac{5}{12}\pi+\frac{\pi}{2}k)+isin(\frac{5}{12}\pi+\frac{\pi}{2}k)|k=0,1,2,3\};\ \{z:\ Re(z)=1\};\ \{0,-\frac{1}{2}\pm\frac{\sqrt{3}}{2}\};\ \{-i\}$
9. $\{cos(\frac{2\pi k}{n})+isin(\frac{2\pi k}{n}):\ k=0,1,...,n-1\}$
10. i. True, ii. True, iii. True. iv. True. v. True. vi. False. vii. True.
11. proof
12. proof
13. i-iii. |z-a|=r Circles with centers at point a, and radius r.
    iv. $\{z:\ Re(z)\leq 3\}$ - half plane.
    v. The empty set.
14.

1	1
2	4
3	5
4	2
5	3
6	6

15. proof
16. proof
17. $i.\ a^{-1} = \frac{5-2i}{27},\ x = \frac{3-7i}{27}\ ii.\ a^{-1} = \frac{-1+2\sqrt{3}}{11},\ x = \frac{-7+3\sqrt{3}}{11}\ iii.\ a^{-1} = \bar{5},\ x = \bar{6}$
18. i. False, ii. False, iii. True.
19. proof
20. proof
21. The real and image part of $\frac{e^{inx}-1}{e^{ix}-1}$ for all $x \neq 0\ (mod\ 2\pi)$.
22. proof