Basic topics in Topology and Geometry

201.2.5221

Midterm exam, two hours

25.12.2018 (D.Kerner)

(1) (23 points) Let $X = \sharp^g(S^1 \times S^1)\sharp^q\mathbb{RP}^2$. Compute $\pi_1(X \setminus \ n\ points)$, for $g, q, n > 0$.

(2) (23 points) Let $X = {}^{[0,1]^2}/_{\{(x,0)\sim(x,1),\ (0,y)\sim(1,1-y)\}}$. Denote the image of the edge $[0,1]\times\{0\}$ by a_1 and the image of the edge $\{0\}\times[0,1]$ by a_2. Does there exist a self-homeomorphism of X that interchanges a_1 and a_2?

(3) (23 points) Does there exist a covering map from $\sharp^g(S^1 \times S^1)$, $g > 1$ to $S^1 \times S^1$?

(4) (23 points) Let $X = S^n \cup S^m \cup S^l$, $n, m, l > 1$, here every two spheres intersect at one point, while the triple intersection is empty. Construct/describe/classify all the path-connected coverings of X.

(5) (23 points) A section of a covering map $\tilde{X} \xrightarrow{p} X$ is a continuous map $X \xrightarrow{s} \tilde{X}$ satisfying $p \circ s = Id$.
Prove: a path-connected covering admits a section iff $\tilde{X} \xrightarrow[\sim]{p} X$.

Good luck!