Introduction to Topology and Geometry

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No auxiliary material is allowed.

The following groups can be assumed known/used freely: $H_*(\mathbb{RP}^n,\mathbb{Z})$, $H_*(\mathbb{CP}^n,\mathbb{Z})$, $\pi_1(S^m)$.

(1) (15 points) Construct/describe the universal covering space of S^2 with two punctures.

(2) (15 points) Compute $H_*(\mathbb{RP}^{2n+1},\mathbb{Z}/_{(3)})$ and $H^*(\mathbb{CP}^{2n+1},\mathbb{Z}/_{(2)})$.

(3) (a) (10 points) Compute $\pi_1(\mathbb{RP}^n)$, $n>1$.

(b) (25 points) Let $\gamma=\{x_0=0\}\subset\mathbb{RP}^2$ (with homogeneous coordinates $[x_0:x_1:x_2]$). Fix an equatorial embedding $\gamma\overset{\tilde\phi}{\hookrightarrow}S^2$, i.e. $\tilde\phi(\gamma)=\{x_3=0\}\subset S^2\subset\mathbb{R}^3$. Prove that $\tilde\phi$ extends to a continuous map $\mathbb{RP}^2\overset{\phi}{\to}S^2$.

(4) (a) (10 points) Describe briefly a map $S^{17}\to S^{17}$ of degree 2019. (No need to go into the deep details, just state the main steps.)

(b) Suppose a locally contractible space X admits the cell structure e^0,e^2,e^3.

(i) (15 points) Suppose $H_*(X,\mathbb{Z})$ is torsion-free. Classify the possible values of the sequence $H_0(X,\mathbb{Z})$, $H_1(X,\mathbb{Z})$, $H_2(X,\mathbb{Z})$,... (Realize each case by an example.)

(ii) (15 points) Suppose $H_*(X,\mathbb{Z})$ has torsion. Classify all the possible values of the sequence $H_0(X,\mathbb{Z})$, $H_1(X,\mathbb{Z})$, $H_2(X,\mathbb{Z})$,... (Realize each case by an example.)

Good Luck!