Introduction to Topology and Geometry

Moed Bet, 21.02.2011.

BGU (201.2.5221) Lecturer: D.Kerner.

No auxiliary material is allowed. Exam duration: 3 hours. Total 105 points.

The following groups can be assumed known/used freely: $H_*(\mathbb{RP}^n;\mathbb{Z})$, $H_*(\mathbb{CP}^n;\mathbb{Z})$, $\pi_1(S^m)$.

(1) (a) (15 points) Fix some embedding $S^1 \subset S^n$, $n \geq 3$. Contract S^1 to a point and denote by X the resulting topological space. Compute $H_j(X;\mathbb{Z})$ for all $j \geq 0$.

(b) (15 points) Compute $H^*(\mathbb{RP}^2 \vee \mathbb{RP}^1; \mathbb{Z}/8)$.

(2) (a) (15 points) Does there exist a covering of S^2 by Klein bottle?

(b) (15 points) Does there exist an embedding $\mathbb{RP}^1 \subset \mathbb{RP}^2$ such that $\mathbb{RP}^2$ retracts onto $\mathbb{RP}^1$?

(3) (15 points) Let X be the union of k lines through the origin in $\mathbb{R}^n$, $n \geq 3$. Compute $\pi_1(\mathbb{R}^n \setminus X)$.

(4) Construct a two-dimensional cell complex X as follows. Put $X^1 = S^1 \vee S^1$. Denote the loops corresponding to these circles by a, b. (So that $[a], [b]$ generate $\pi_1(S^1 \vee S^1)$.) Now glue the cells e^2_a, e^2_b, e^2_{ab} by the attaching maps $\partial e^2_a = a^2$, $\partial e^2_b = b^3$, $\partial e^2_{ab} = abab^2$.

(a) (15 points) Compute $\pi_1(X)$.

(b) (15 points) Does X admit a non-normal covering?

Good Luck!