Computing AC losses in stacks of superconducting tapes

Leonid Prigozhin and Vladimir Sokolovsky

Ben-Gurion University of the Negev, Israel

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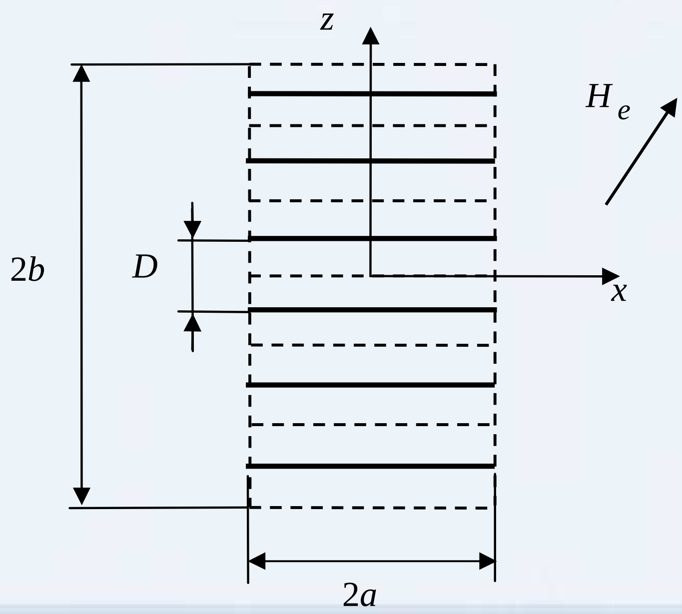
stacks of type-II sc tapes

Simplified models of Roebel cables and tape coils: stacks of parallel sc tapes, each carrying the same transport current

Previous works:

1. **Stacks of several strips:**
   Pardo, Sanches, Navau 03, Grilli et al. 06, Pardo 08, Brambilla et al. 09, …
   numerical simulation
2. **Infinite stacks of thin tapes: Mawatari 96, Muller 97:**
   analytical solutions, the Bean model
3. **Finite stacks of densely packed thin tapes:**
   Clem, Claassen, Mawatari 07:
   anisotropic bulk sc limit, approx. solution
   Yuan, Campbell, Coombs 09,10:
   extension of the Clem et al. model



this talk

Our aim: to refine the bulk model, to study convergence of stack problems to the bulk limit, to improve and simplify the AC loss computation

1. **Numerical solution of stack magnetization and transport current problems:** we investigate the peculiarities of solution to stack problems

2. **Modification of the Clem *et al.* anisotropic bulk sc model:** instead of an assumption about the current density in the subcritical zone (*solution property*), we postulate zero conductivity in normal to tapes direction for the bulk approximation (*anisotropic bulk media property*)

3. **A free-marching numerical algorithm for the bulk problems:** existing front tracking schemes are not universal; we also compute the current density and electric field simultaneously to determine the AC losses directly (for any critical-state model).

4. **Checking accuracy of the anisotropic bulk approximation:** fast convergence to the bulk limit

5. **Conclusion**

stack magnetization problems

Our approach (a modification of the method in *LP, IEEE Trans on ASC,* 1997**):**

A-V formulation, $E_i(x,t) = -\partial_t \left(A[J] + A_e \right)\Big|_{\Gamma_i} - C_i(t)$ **in each tape** Γ_i;

here $J = \left(J_1(x,t), \ldots, J_N(x,t) \right), \quad C_i = \partial V / \partial y \big|_{\Gamma_i}$.

A nonlinear current-voltage relation for the superconductor.

We assume a <u>critical-state model</u> with the field-dependent critical sheet current density $| J_i(x,t) | \leq J_c(\bar{H}), \quad i = 1, \ldots, N, \quad -a < x < a, \quad 0 < t$

and define the set *K* of <u>test vector functions</u> J' **also satisfying these constraints.**

Since the magnetic field is the sum of external field and that induced by the tape currents, $\bar{H}[J] = \bar{H}_e + \bar{H}_i[J],$ **the set depends on the unknown solution:**

$$K[J] = \left\{ J' = \left(J_1'(x,t), \ldots, J_N'(x,t) \right) \ : \ | J_i' | \leq J_c(\bar{H}[J]), \quad i = 1, \ldots, N \right\}$$

It follows from the critical-state model that $E_i(J_i - J_i') \geq 0$ **for any** $J' \in K[J].$

The result is a <u>variational inequality</u>.

quasi-variational inequality for stacks:

Find $C = (C_1(t), \ldots, C_N(t))$ and $J \in K[J]$ such that $J|_{t=0} = J^0$,

$$\sum_{i=1}^{N} \left\{ \int_{-a}^{a} \left(\partial_t \left(A[J] + A_e \right)|_{\Gamma_i} + C_i \right) \left(J_i' - J_i \right) dx \right\} \geq 0 \quad \text{for any } J' \in K[J]$$

and also $\int_{-a}^{a} J_i(x,t)\,dx = I_{tr}(t), \qquad i = 1, \ldots, N.$

Here the functions $C_i(t)$ are determined implicitly by the transport current in each tape, A_e is the vector potential related to the external field, and

$$A[J](x,z,t) = -\frac{\mu_0}{2\pi} \sum_{i=1}^{N} \int_{-a}^{a} J_i(x',t) \ln\left(\sqrt{(x-x')^2 + (z-z_i)^2} \right) dx'$$

is the vector potential of tape currents. Electric field in the tapes :

$$E_i = -\partial_t \left(A[J] + A_e \right)\Big|_{\Gamma_i} - C_i$$

implicit discretization

On each time layer $t^m = m\tau$ the inequality is equivalent to the following <u>implicit</u> constraint optimization problem

$$\text{Find } C^{m-1/2} \text{ and } J^m \text{ such that}$$

$$J^m = \mathop{\arg\min}_{J \in K[J^m]} F\left(J, C^{m-1/2}\right)$$

$$\text{and } \int_{-a}^{a} J_i^m dx = I_{tr}(t^m), \quad i = 1, \dots, N,$$

where $F(J,C) = \sum_{i=1}^{N} \left\{ \int_{-a}^{a} \left(\frac{1}{2} A[J]|_{\Gamma_i} + \left(A_e^m - A_e^{m-1} - A[J^{m-1}] \right)|_{\Gamma_i} + \tau C_i \right) \cdot J_i \, dx \right\}.$

Electric field is determined at the intermediate layer:

$$E_i^{m-1/2} = -\frac{1}{\tau} \left(A[J^m] + A_e^m - A[J^{m-1}] + A_e^{m-1} \right)\Big|_{\Gamma_i} - C_i^{m-1/2}$$

We used piece-wise constant finite elements for J_i and E_i in space and accurately approximated the integrals (some of them are singular).

iterative solution

Main idea: simultaneous resolution of transport current constraints and the dependence $K=K(J)$ caused by $J_c=J_c(H)$.

The iterations to find J^m and $C^{m-1/2}$:

(P) $$J^{(n+1)} = \arg \min_{J \in K[J^{(n)}]} F_\lambda(J, C^{(n)})$$

$$C_i^{(n+1)} = C_i^{(n)} + \rho \left(\int_{-a}^{a} J_i^{(n+1)} dx - I_{tr}(t^m) \right), \quad i = 1, \ldots, N,$$

where n is the iteration number, ρ, λ are positive coefficients, and

$$F_\lambda(J, C) = F(J, C) + \lambda \sum_{i=1}^{N} \left(\int_{-a}^{a} J_i dx - I_{tr}(t^m) \right)^2$$

is an augmented functional: the additional term helped to obtain convergence.

On each iteration the explicitly-constrained optimization problem (P) was solved efficiently using a simple under-relaxation with projection algorithm.

simulation

We assume the Kim model $J_c(\bar{H}) = \dfrac{J_{c0}}{1 + \dfrac{\sqrt{k^2 H_x^2 + H_z^2}}{H_0}}$ with

k=1 (isotropic case) or k=0 (anisotropic case) and use dimensionless variables:

$$(x^*, z^*) = (x, z)/2a, \quad J^* = J/J_{c0}, \quad H^* = H/J_{c0}, \quad H_0^* = H_0/J_{c0},$$

$$I_{tr}^* = I_{tr}/2aJ_{c0}, \quad E^* = E/\left(2a\mu_0 f J_{c0}\right), \quad P^* = P/\left(4a^2\mu_0 J_{c0}^2\right)$$

The typical values of normalizing factors:

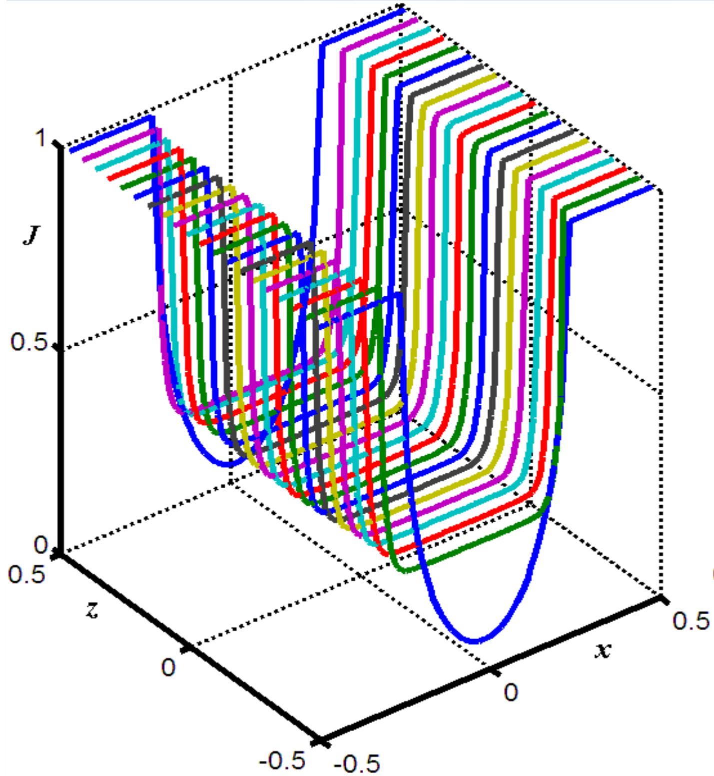
$$E_0 \simeq 10^{-2} - 10^{-1} V/m, \quad P_0 \simeq 10^{-1} - 10^0 J/m$$

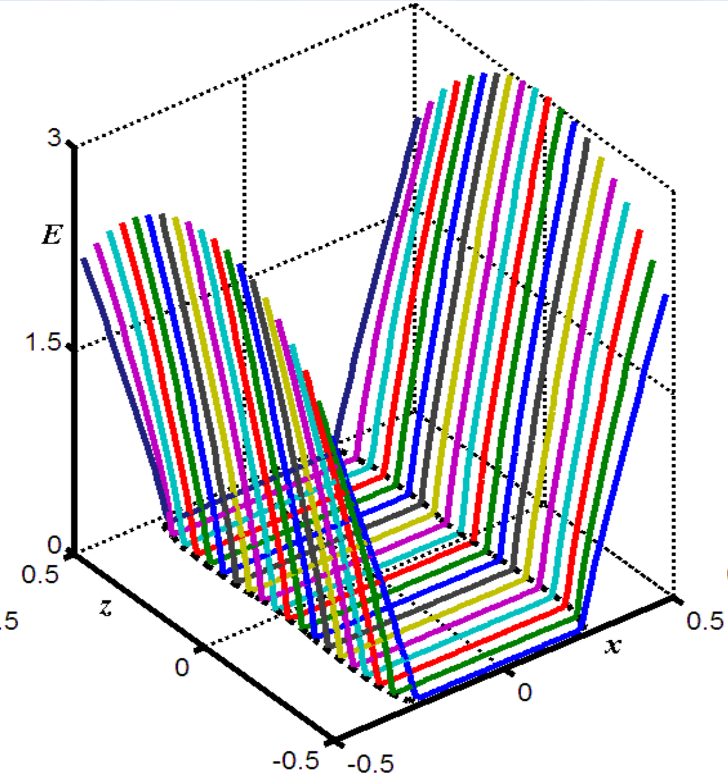
We consider first the special case corresponding to the Bean model: $H_0 = \infty$

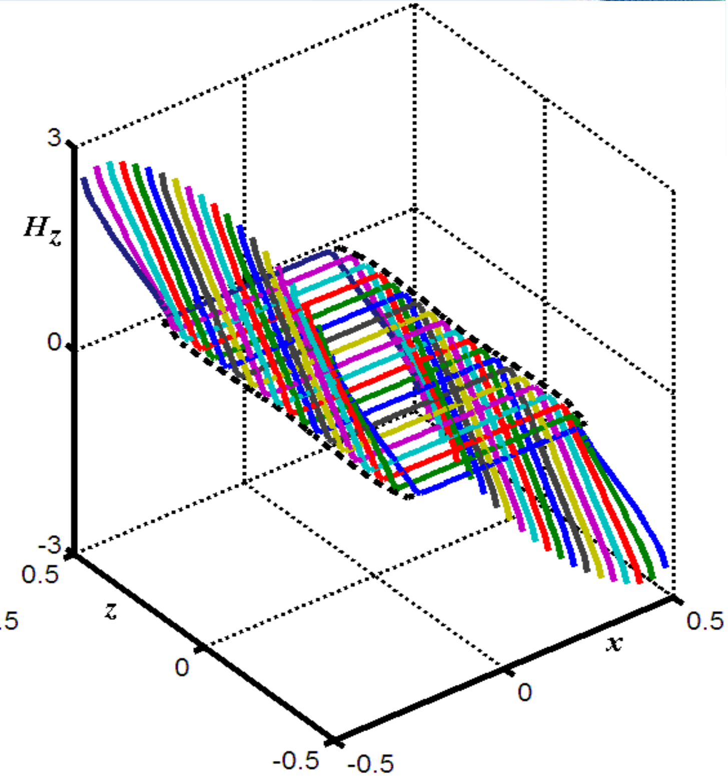
This case is instructive and easier to understand.

Bean’s model, applied transport current

sheet current densities | **electric field** | **the normal magnetic field**

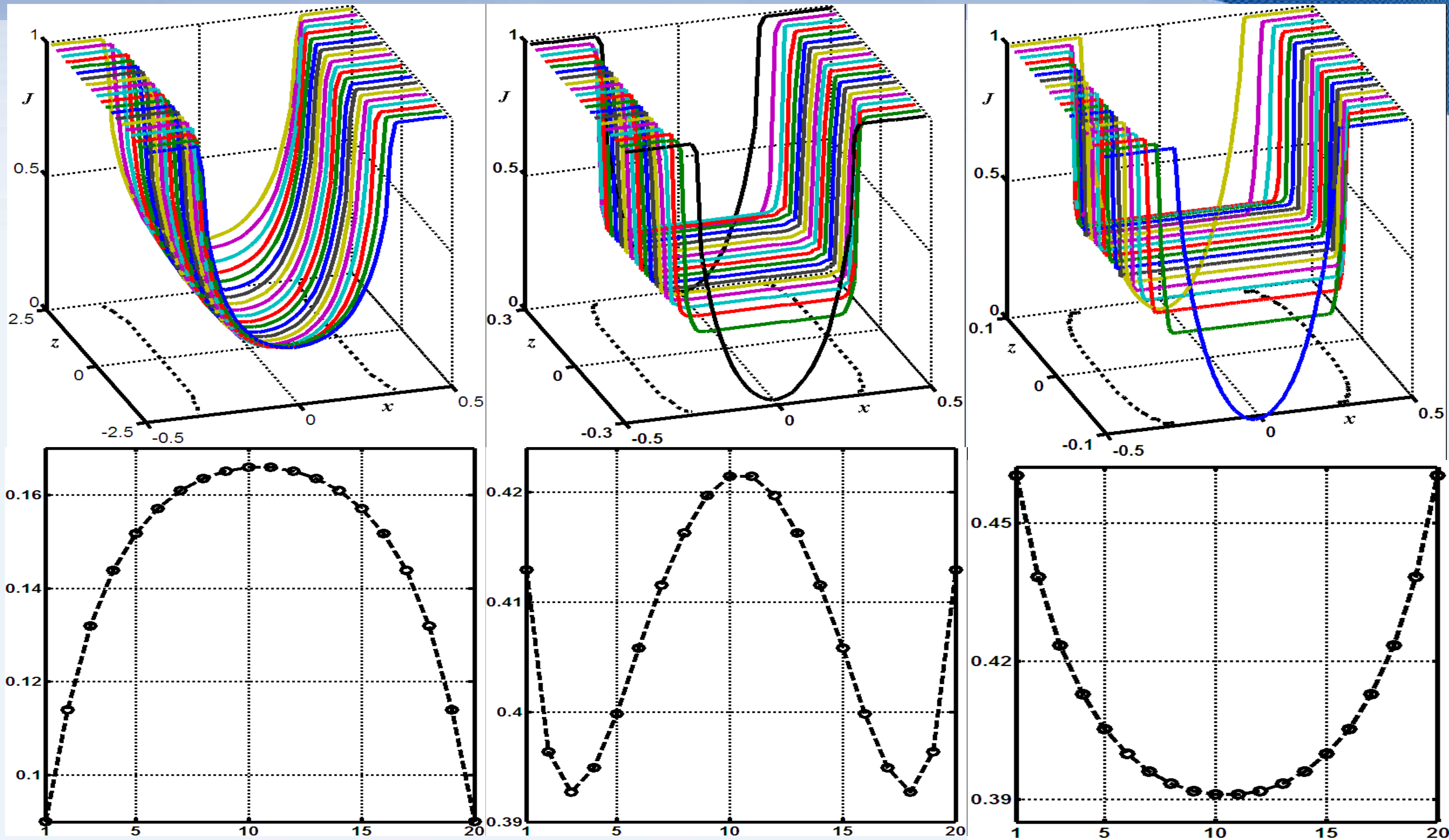






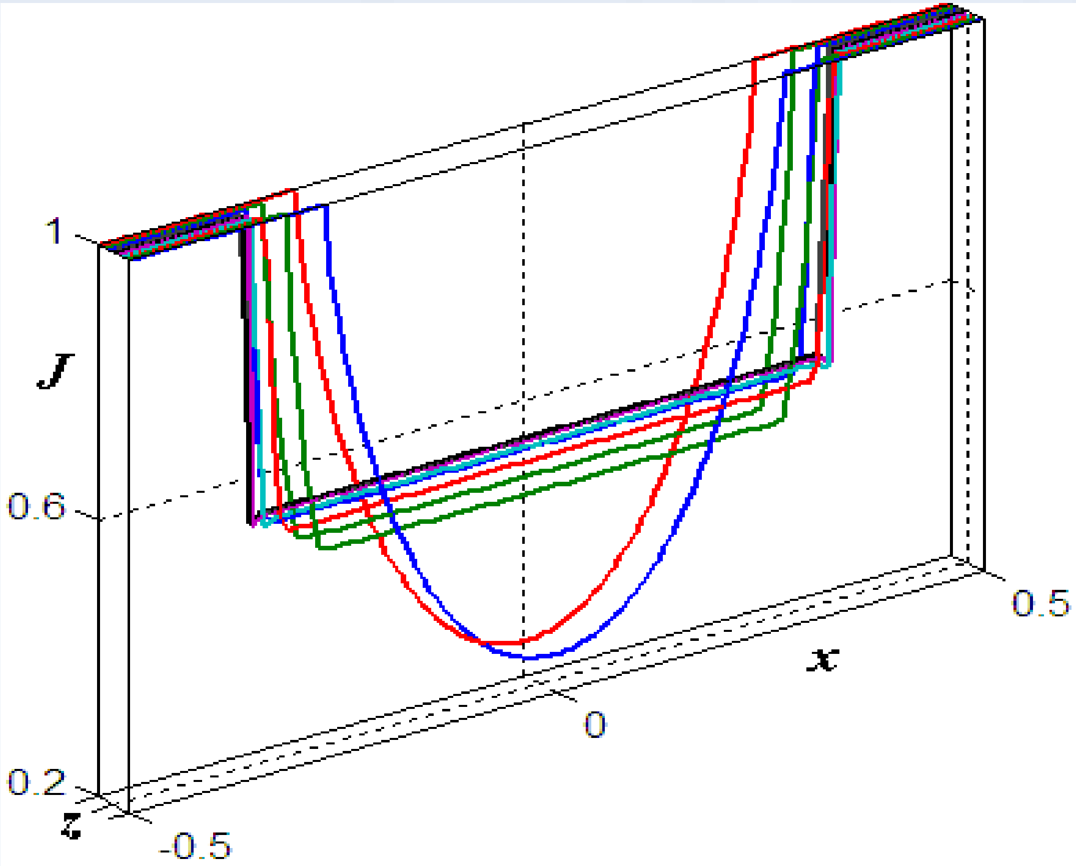
A 20-tape stack, stack height = tape width, Bean’s model, the transport current case; simulation results for $I_{tr} = 0.6$. **The dotted line indicates the critical zone boundary. In the subcritical zones of all tapes both electric field and the normal magnetic field component are zero with high accuracy. The current densities in these zones are close to constant (as was assumed in previous works) in all except the top and bottom tapes.**

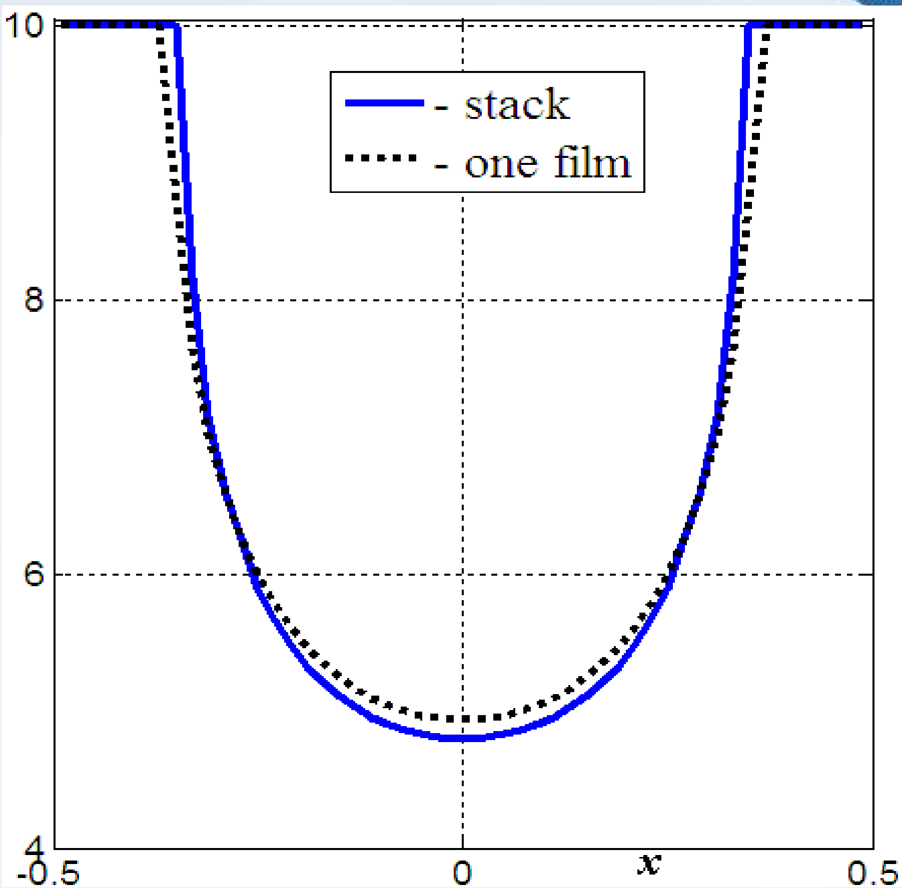
Bean's model, $I_{tr} = 0.7\sin(2\pi t)$



Current densities J_i at t=0.16 and AC losses per period P_i in tapes of a 20-tape stacks of different heights. From left to right: *D*=0.25, 0.03, 0.01. Dotted lines show the critical zone boundaries. The shape of these zones changes with *2b*.

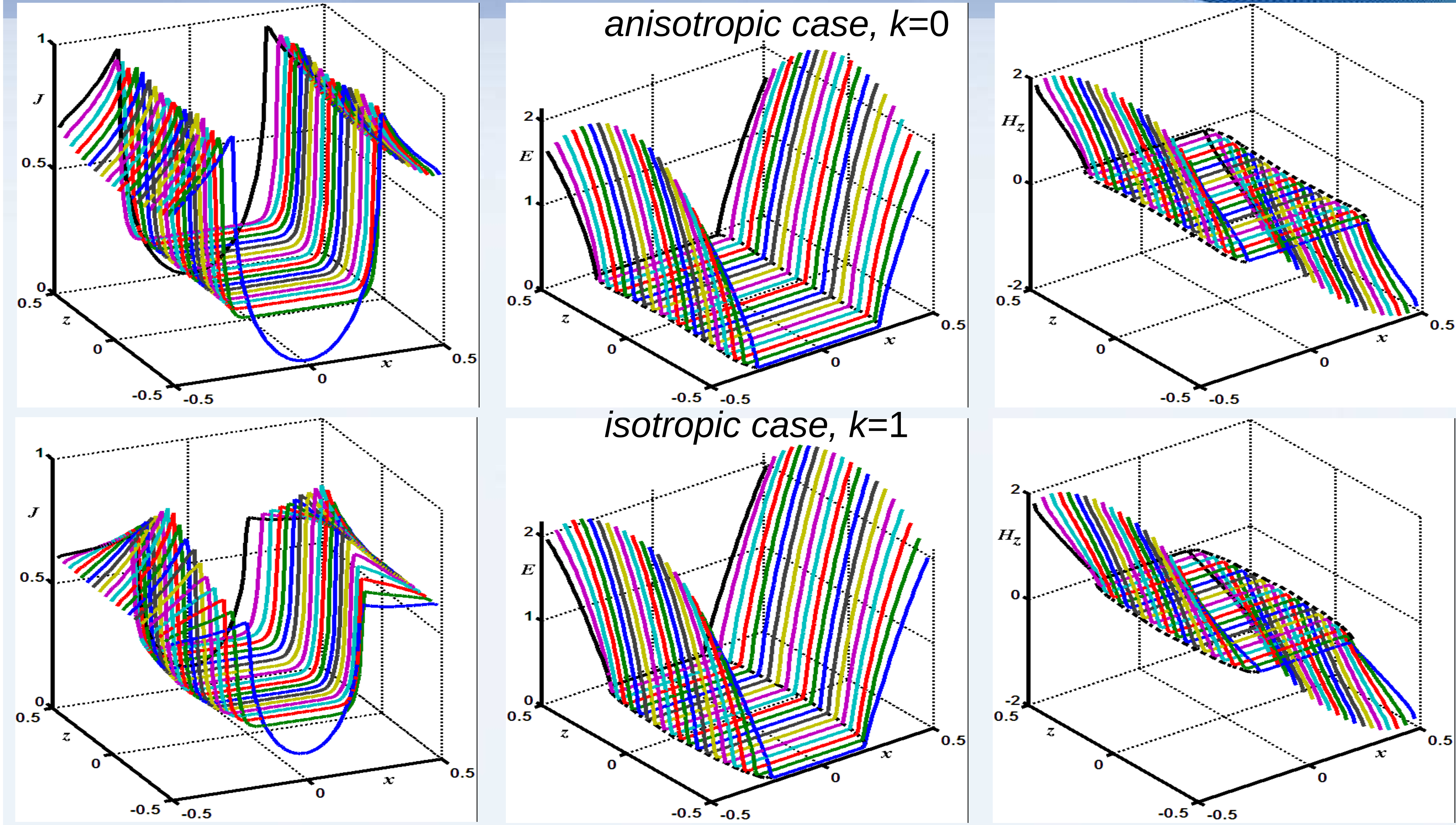
comparison of a very thin stack and an isolated tape





Thin 10-tape stack, the Bean model, D=0.005, I_{tr} = 0.7. Sheet current densities in the stack tapes (left) are not similar to the known distribution of current in an equivalent isolated thin film [Brandt and Indenbom, 93]. However, the sum of these densities (right, solid line) converges to this analytical solution (dotted line). The total AC losses converge as well.

the Kim model



20-tape stack, stack height = tape width, H_0=4, $I_{tr} = 0.5\sin(2\pi t)$.

Shown: J, E, H_z in tapes for t=0.18. Top row: k=0, $J_c(|H_z|)$; bottom row: k=1, $J_c(|H|)$.

CPU time to compute AC losses per period with ~1% accuracy is about 3 minutes.

anisotropic bulk model

Solving stack probems is time consuming if the number of tapes ***N* ~ 10²**.
For densely packed stack of many tapes Clem, Claassen and Mawatari 07 suggested an anisotropic bulk approximation. The model was further extended in a series of works by Yuan, Campbell and Coombs 09,10.

We avoid the simplifications made in these works to find an approximate solution.
Let a long bulk sc with the cross-section [-a,a]x[-b,b] in XZ plane be placed into the external field H_e. We assume zero conductivity in z-direction (normal to the tapes) and a critical-state model $E(j)$ relation for the bulk current density with the critical value

$$|j| \le j_c(\bar{H}) = \frac{j_{c0}}{1 + \frac{\sqrt{k^2 H_x^2 + H_z^2}}{H_0}}$$

Since there is no conductivity in the *z*-direction, we can postulate that for any $-b < z < b$

$$\int_{-a}^{a} j(x,z,t)\,dx = i_{tr}(t)$$

This becomes a **<u>homogenized stack problem</u>** if $J_c = j_{c0} D,\ I_{tr} = i_{tr} D,$

where $D=2b/N$ is the inter-tape distance.

In particular, it is supposed that for $D << a$ the AC loss in a stack should be close to that in the anisotropic bulk sc. We checked this hypothesis numerically.

anisotropic bulk model

Quasi-variational inequality:

Find C=$C(z,t)$ and $j \in K_0[j]$ such that $j|_{t=0} = j^0$,

$$\int_{-b}^{b}\int_{-a}^{a}\left(\partial_t\left(A[j]+A_e\right)+C\right)\left(j'-j\right)dx\,dz \geq 0 \quad \text{for any } j' \in K_0[j]$$

and also $\displaystyle\int_{-a}^{a} j(x,z,t)dx = i_{tr}(t)$ for every $z \in [-b,b]$.

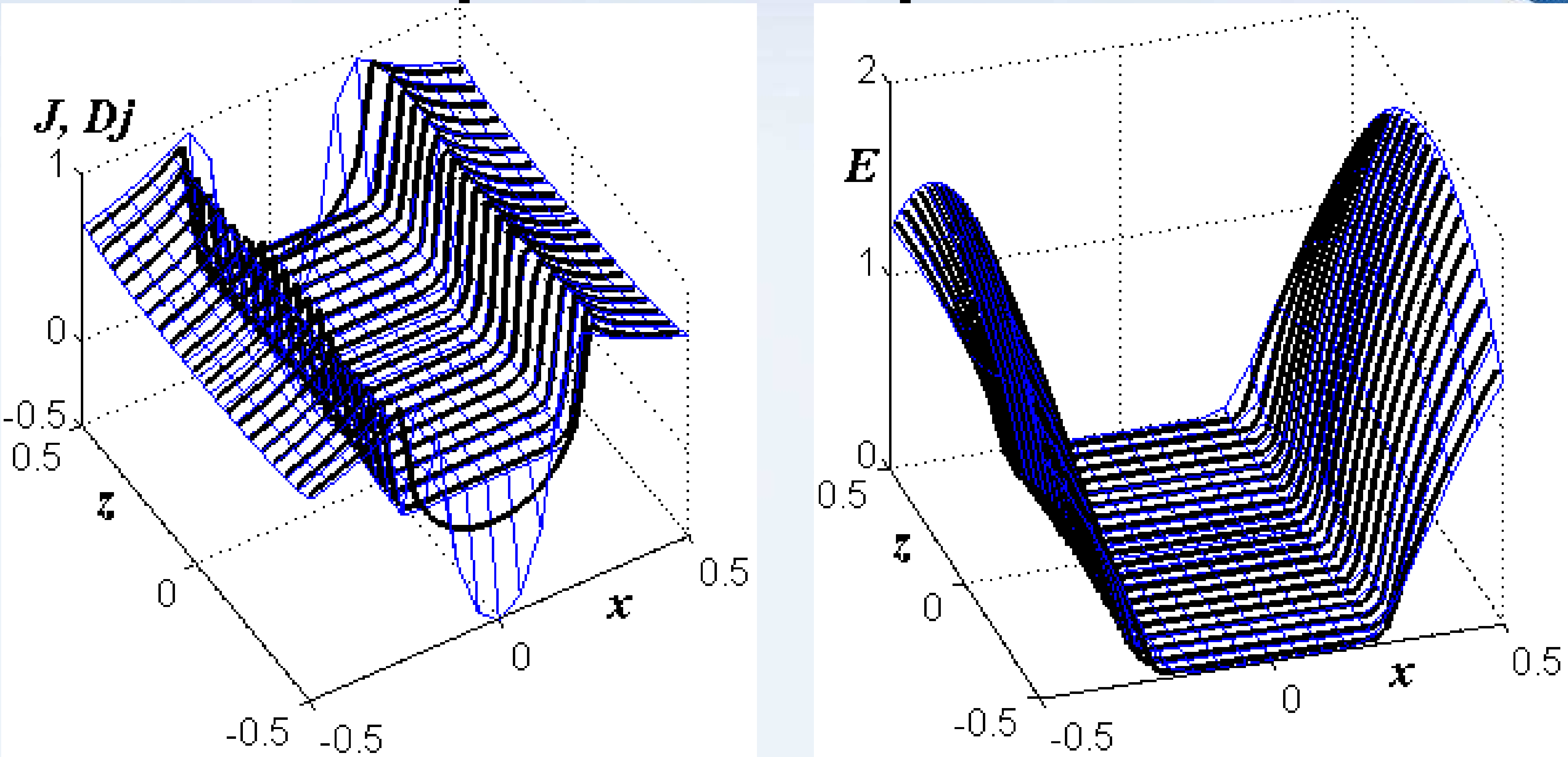
The variational formulation and numerical scheme are very similar to those for stacks. Electric field is determined directly also in this case,

$$E(x,z,t) = -\partial_t\left(A[j]+A_e\right) - C(z,t),$$

so the AC loss computation is again straightforward.

No assumptions about the critical zone shape is needed.

comparison of solutions:
a transport current problem



20-tape stack, stack height = tape width, *D*=0.05. Kim's model, H_0=4, *k*=0 (anisotropic case).

Shown for I_{tr}=0.5:

Left - current densities in tapes *J* (black) and the rescaled bulk one, *Dj* (blue).

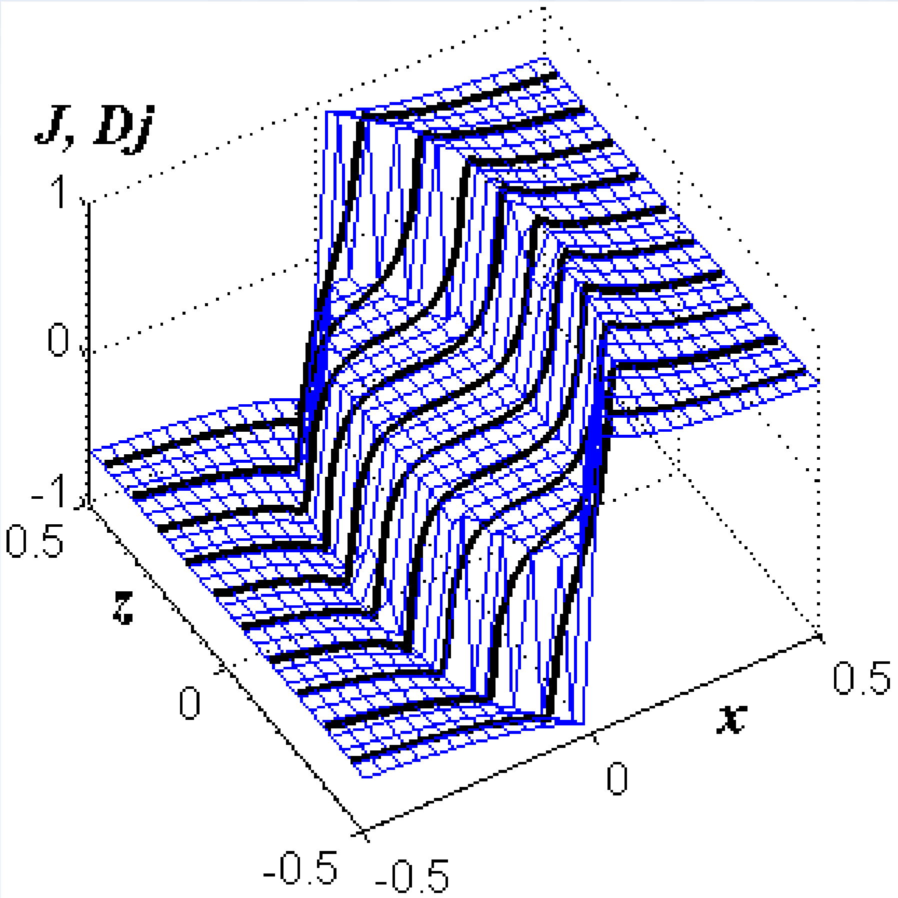
Right – electric field in tapes (black) and in bulk (blue).

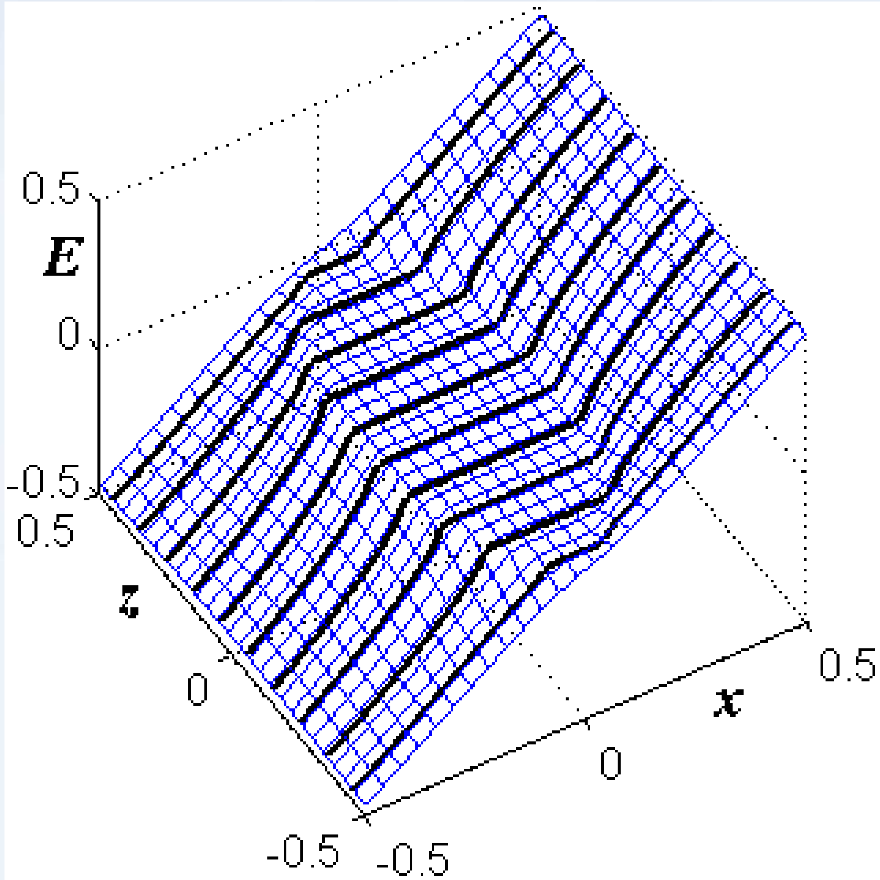
accuracy of the bulk approximation in AC loss-per-period estimates

For $I_{tr}(t) = 0.5\sin(2\pi t)$ **and both iso- and anisotropic Kim models (*H_0*=4):**

the number of tapes N	stack height *2b* (in tape width units)	*D=2b/N*	error, $\Delta P / P$
20	1	0.05	0.7%
10	1	0.1	4%
10	0.5	0.05	1.5%
10	0.25	0.025	0.5%

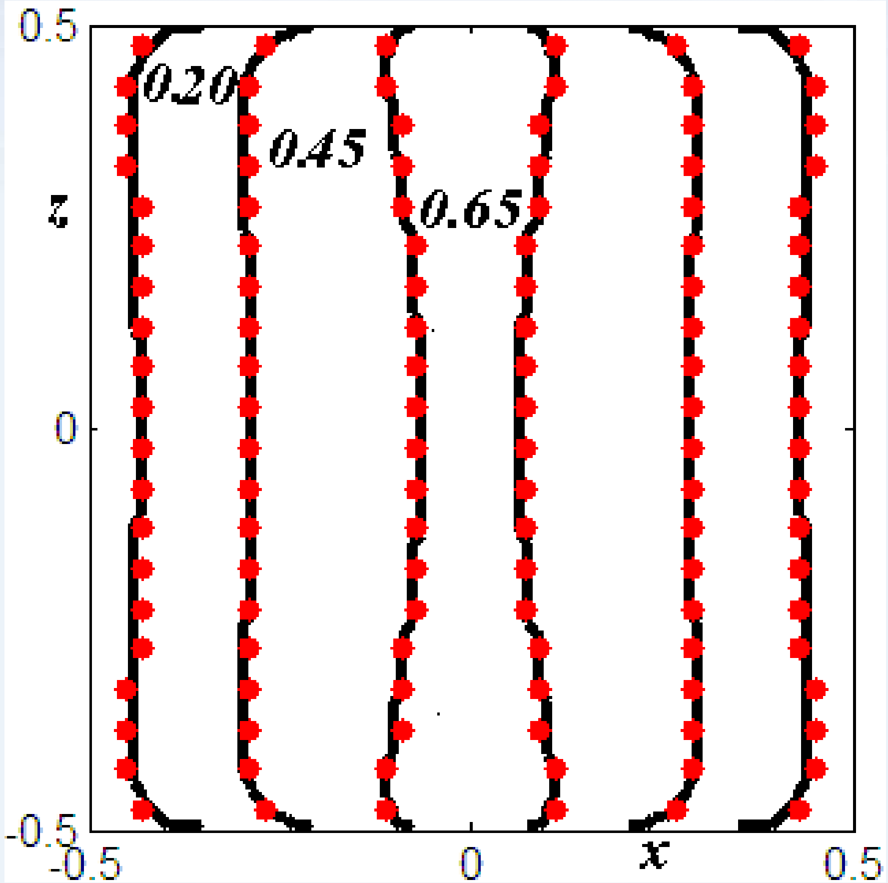
comparison of solutions: a magnetization problem

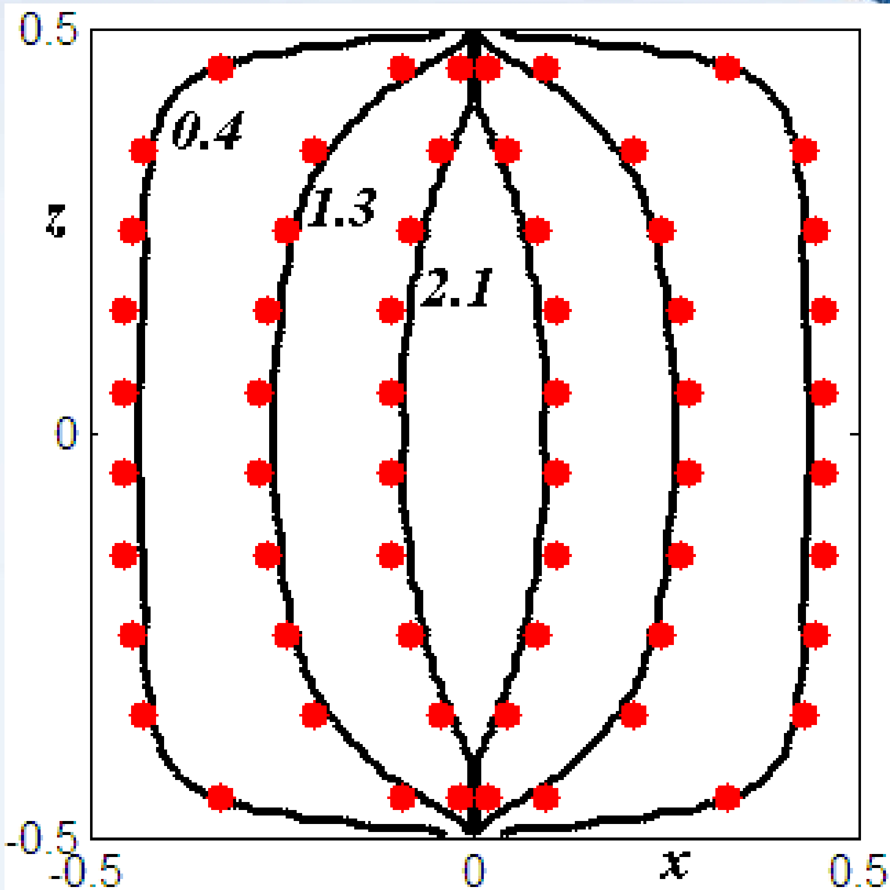




10-tape stack, stack height = tape width, D=0.05. Kim's model, H_0=4, k=1.
Shown for H_{ez}=1.7:
Left - current densities in tapes J (black) and the rescaled bulk one, Dj (blue).
Right – electric field in tapes (black) and in bulk (blue).
Good agreement despite the small number of tapes (especially, in the critical zone, where all the losses occur). Solutions for ani- and isotropic bulk sc are the same.

propagating critical current zone, in stack and anisotropic bulk sc models





Shrinking subcritical core boundaries: in tapes (...) and in the bulk (——).

Left: a 20-tape stack (Kim, *k*=0) carrying growing transport current, I_{tr}=0.20, 0.45, 0.65**;**

Right: a 10-tape stack (Kim, *k*=1) in a growing external field, $H_{e,z}$=0.4, 1.3, 2.1

The free boundary topology changes in the right plot. For $H_{e,z}(t) = 2.5\sin(2\pi t)$ the losses in a 10-tape stack and the bulk sc are 5.74 and 5.95, resp., the error ~3.5%

CONCLUSIONS

1. **Our numerical scheme is efficient for estimating AC losses in stacks of several tens of thin tapes.**
2. **For densely packed stacks of many tapes, the anisotropic bulk approximation proposed by Clem, Claassen and Mawatari is preferable. We improved this approximation and derived an efficient free marching numerical scheme also for the bulk problems.**
3. **Numerical simulations of transport current and magnetization problems for properly scaled stack and bulk sc models confirmed convergence of stack problems solution to the anisotropic bulk limit.**
4. **An alternative approach is to replace the stack problem with many tapes by appropriately scaled simpler problems for stacks of the same height and gradually increase the number of tapes until AC losses converge. The convergence is fast: in our simulations the error was less (usually much less) than 2% for *D/2a* ~ 0.025.**

Thank you!

L. Prigozhin and V. Sokolovsky, arXiv:1101.5055v1