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AC losses in thin coated conductors under non-sinusoidal conditions

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Abstract
AC losses in superconducting wires and tapes are usually studied for applied sinusoidal currents and/or magnetic fields. However, currents in electric power systems contain a variety of harmonics. We solved analytically and numerically, in the infinitely thin approximation, the transport current and magnetization problems for coated conductors under non-sinusoidal conditions. The analytical expressions for eddy current and hysteresis losses have been obtained in the framework of the critical state model neglecting the response of the normal-metal substrate and stabilization layers. The contribution of higher harmonics to losses per cycle is determined by both their phase shift relative to the main harmonic and their amplitude. It has been shown that the 5% third current harmonic (for the phase shift π) increases eddy losses in the normal-metal parts by up to 90% at a transport current close to the critical value.

Numerically, for the power law current–voltage characteristic of a superconductor, the contribution of higher harmonics to the total losses in a coated conductor was investigated in a wide range of the power index. It has been shown that even at a low power index ($n = 4$) this contribution can achieve 44% of losses caused by the main harmonic only. For high external magnetic fields an approximate analytical solution has also been derived and compared to the numerical solution.

(Some figures may appear in colour only in the online journal)

1. Introduction

Second generation (2G) $YBa_2Cu_3O_{6+x}$ (YBCO) coated conductors are currently considered the main candidate for broader commercialization of HTSC wires [1, 2]. Designing superconductors with low losses under AC conditions (AC transport current or/and magnetic field) is most important for their implementation in transformers, motors, generators, etc. Under different conditions, AC losses in coated conductor tapes and coils have been studied in a vast number of works (see e.g. [3–5] and references therein). However, most of these works consider sinusoidal magnetic fields or/and currents [5–7]. In reality, currents in electric power systems contain a variety of harmonics. The amplitudes and phases of higher harmonics are determined by such factors as the rated voltage, type of power source, load type, connection type of transformer windings (the Δ- or Y-connection), method of higher harmonic elimination, etc (see, e.g., [8–10] and references therein). In the general case the spectrum can contain a variety of harmonics with different amplitudes and phases

$$I(t) = \sum_k I_k \cos(k\omega t + \phi_k). \quad (1)$$

Here I_k and ϕ_k are the amplitude and phase of the kth harmonic, respectively, $k = 0, 1, 2, \ldots$, and $\omega = 2\pi f$, where f is the first harmonic frequency. Typically, in power systems, this frequency is 50 or 60 Hz; for special electric systems (airplanes, ships, etc) f can be 400–800 Hz. Below, we always assume $\phi_1 = 0$.

In AC power systems $I_0 = 0$ (I_0—direct current) and, according to standard requirements for the quality of power supplying, higher harmonics should not exceed a few per cent of the first one. The losses in normal conducting parts of power devices are usually determined by the main harmonic. However, converters, non-linear reactors, transient and overload conditions make currents strongly non-sinusoidal: the losses due to the higher harmonics can increase and higher harmonics have to be taken into account for the loss estimation. Since superconductors possess a strongly non-linear current–voltage characteristic, one can expect a substantial contribution of higher harmonics to AC losses in superconducting elements. Recently, it has been shown [11, 12] that the contribution of higher harmonics to AC losses in superconducting bulk and thin film samples can be greater by more than an order of magnitude than in normal-metal samples of the same shape. For example, the 5% third harmonic of a low magnetic field can increase the losses by 20% in a superconducting strip [11], while in normal-metal conductors the contribution of this harmonic is about 2%. We note, however, that these losses depend on the phases ϕ_k: in a certain range of the phases the odd harmonics can reduce the total losses.

In this paper we study the AC losses in a coated conductor under non-sinusoidal conditions. We consider two practically important cases: (i) a non-sinusoidal external magnetic field H_e applied perpendicular to the conductor wide surface and (ii) non-sinusoidal transport current in the conductor in zero external magnetic field. Assuming that both the superconducting and normal-metal layers are thin, we take into account only the losses caused by magnetic flux perpendicular to them moving in and out of the tape edges; the losses caused by penetration of a parallel flux from the coated conductor top and bottom surfaces are neglected. Using the critical state model with a field-independent critical current, we derive analytical expressions for the losses in the superconducting and normal-metal parts of a coated conductor (section 2). We also present the results of numerical simulation for superconductors with a power law current–voltage characteristic (section 3). For high external magnetic fields an approximate analytical solution is derived and compared to the numerical solution.

2. AC power losses in coated conductors: an analytical approach

Coated conductors are produced by the deposition of a 1–3 μm $YBa_2Cu_3O_{6+x}$ film upon a 10–100 μm thick metallic substrate; the superconducting film is subsequently covered by protective silver and stabilizing copper layers. The total AC losses in a coated conductor (figure 1) are a sum of the losses in its superconducting and normal-metal parts. For simplicity, we assume that the thicknesses of superconducting and normal-metal layers, d_{sc} and d_m, respectively, are much less than the conductor width $2a$ and the conductor is infinitely long.

Let the superconductor be described by the critical state model with a field-independent critical current density. To estimate AC losses analytically, we neglect the magnetic flux of the induced current in a normal-metal strip in comparison with the total magnetic flux (of the external magnetic field and that produced by a current in a superconducting layer). For a

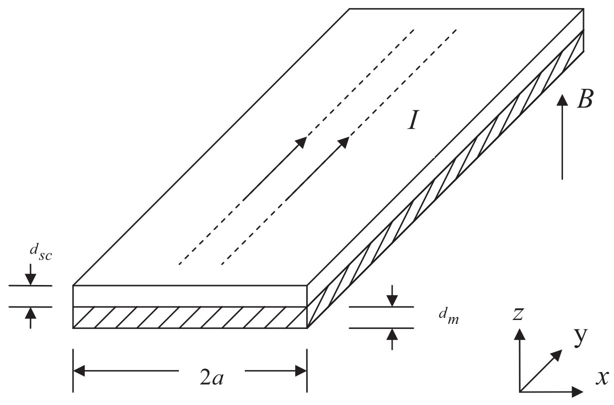

Figure 1. Sketch of a coated conductor.

sinusoidal magnetic field perpendicular to the normal-metal strip, such an approximation is appropriate (see [2, 13]) if

$$\frac{\mu_0 a d_m \omega}{\pi \rho} \ll 1, \tag{2}$$

where μ_0 is the magnetic permeability of vacuum and ρ is the metal resistivity; the inverse quantity, $\alpha = \frac{\pi\rho}{\mu_0 a d_m \omega}$, will be used in this work. We replace the substrate, protective silver and stabilizing copper layers by an effective normal-metal strip with d_m/ρ equal to $d_{sub}/\rho_{sub} + d_{sil}/\rho_{sil} + d_{st}/\rho_{st}$ (here ρ_{sub}, ρ_{sil}, ρ_{st}, and d_{sub}, d_{sil}, d_{st} are the resistivities and thicknesses of the substrate, protective and stabilizing layers, respectively). We assume that condition (2) is valid for each harmonic of a non-sinusoidal magnetic field that needs to be taken into account.

At consideration of the magnetization problem we assume that a superconductor is influenced by the external magnetic field only. The normal-metal strip is thus subjected to the applied field and to the field produced by a current in the superconductor. Using the thin film approximation, we replace real distributions of current in both metal and superconducting strips by the sheet currents [2, 3, 7, 14].

For transport current problems we assume that the current in the superconductor does not exceed the critical value, $I_c = 2aJ_c$ and the magnetic field induced by a current in the normal-metal layers is also negligible in comparison with the field of the current in the superconductor (here $J_c = j_c d_{sc}$ is the sheet critical current density; j_c is the critical current density). Eddy current losses in the normal metal are induced by variations of the magnetic field produced by a current in a superconducting layer.

2.1. Losses in the superconductor layer

A non-sinusoidal current (magnetic field) can non-monotonically increase from its minimum to maximum or decrease from the maximum to minimum; this leads to appearance of 'additional cycles'. However, additional losses, caused by these cycles, should be taken into account only for relatively high amplitudes of higher harmonics [11]. Thus, for an incomplete magnetic field penetration into a superconducting slab, the contribution of the additional cycles does not exceed 5% even if $H_2/H_1 = 1$; here H_k is the amplitude of the kth harmonic of the external magnetic field.

For periodic external magnetic fields and transport currents varying monotonically between their extreme values, the AC losses in a superconducting strip were recently considered in [11, 12]. Under non-sinusoidal conditions the waveforms of currents (external magnetic field) are asymmetrical—the current maximum is not equal to the modulus of its minimum. It has been shown that the expressions for the loss power in a thin strip per unit of length, obtained for symmetrical waveforms in [14–16] using the Bean critical state model, can be modified for an asymmetrical case as follows.

In a perpendicular magnetic field

$$P_H = 4\mu_0 a^2 f J_c H_c \Delta h g(\Delta h), \tag{3}$$

and in a strip carrying a transport current

$$P_I = \frac{\mu_0 f I_c^2}{\pi} q(\Delta i), \tag{4}$$

where

$$g(\Delta h) = \frac{2}{\Delta h}\ln(\cosh(\Delta h)) - \tanh(\Delta h) \quad \text{and} \quad \Delta h = \frac{H_{max} - H_{min}}{2H_c},$$

$$q(\Delta i) = (1-\Delta i)\ln(1-\Delta i) + (1+\Delta i)\ln(1+\Delta i) - \Delta i^2 \quad \text{and} \quad \Delta i = \frac{I_{max} - I_{min}}{2I_c},$$

$H_c = J_c/\pi$; H_{max}, H_{min}, and I_{max}, I_{min} are the maximum and minimum values of the periodic external magnetic field H_e and transport current I, respectively.

2.2. *Losses in the normal-metal layers and total losses: a non-sinusoidal external magnetic field*

The normal-metal strip of a coated conductor is subjected to a periodic non-sinusoidal non-uniform magnetic field penetrating through the superconducting layer. The z-component H_z of this field at $z = 0$ can be written as

$$H_z = H_z(x, H_{max}, J_c) - H_z(x, H_{max} - H_e, 2J_c), \tag{5}$$

$$H_z = H_z(x, H_{min}, J_c) + H_z(x, H_{min} - H_e, 2J_c), \tag{6}$$

for the increasing and decreasing external field, respectively, see [9, 11]. Here

$$H_z(x, H_e, J_c) = \begin{cases} H_c \operatorname{arctanh}\dfrac{\sqrt{x^2 - b^2}}{c|x|}, & b < |x| < a \\ 0, & |x| < b, \end{cases}$$

$$b = \frac{a}{\cosh(H_e/H_c)}, \qquad c = \tanh(H_e/H_c).$$

Only the y-components of the current density j and electric field E are non-zero; these components obey the Ohm law, $E = \rho j$ (here and below the index y is omitted to simplify the notations). The power of eddy current losses p_{ed} per unit of volume is

$$p_{ed} = j(x, t)E(x, t). \tag{7}$$

Here the electric field can be determined as

$$E(x, t) = -\mu_0 \int_0^x \frac{\partial H_z(u, t)}{\partial t}\, du. \tag{8}$$

Integrating, we find the eddy current loss power P_{Hed} per unit of length

$$P_{Hed} = \mu_0^2 \frac{\omega}{2\pi}\frac{d_m}{\rho}\int_0^T dt \int_{-a}^{a} dx \left[\int_0^x \frac{\partial H_z(u, t)}{\partial t}\, du\right]^2 = \frac{\mu_0^2 d_m a^3 \omega^2}{3\pi\rho} H_1^2 F_h, \tag{9}$$

where

$$F_h = \frac{1}{\omega\pi}\left\{\int_{t_1}^{t_2}\left(\frac{dh_e}{dt}\right)^2\left[1 - \frac{3}{\cosh^2(Y)} + \frac{2}{\cosh^3(Y)}\right]dt + \int_{t_2}^{T+t_1}\left(\frac{dh_e}{dt}\right)^2\left[1 - \frac{3}{\cosh^2(X)} + \frac{2}{\cosh^3(X)}\right]dt\right\},$$

$$h_e = \frac{H_e(t)}{H_1}, \qquad Y = \frac{H_{max} - H_e}{2H_c}, \qquad X = \frac{H_{min} - H_e}{2H_c}, \qquad T = 1/f.$$

Here T is the period; t_1 and t_2 are the time moments at which the external field reaches its maximum and minimum, respectively.

The normalized total loss power (a sum of the loss powers in a superconductor and normal-metal layers) per unit of coated conductor length,

$$p_h = \frac{P_{Htot}}{(\mu_0^2 d_m a^3/3\pi\rho)H_1^2\omega^2} = F_h + \alpha Q_h, \tag{10}$$

where $Q_h = 6\frac{\Delta h}{h_1^2} g(\Delta h)$, $h_1 = \frac{H_1}{H_c}$, F_h and αQ_h are the normalized losses in the normal-metal and superconducting strips, respectively. Here the total losses $P_{Htot} = P_H + P_{Hed}$ are normalized by the loss in the normal-metal strip without the superconductor placed in a uniform sinusoidal magnetic field having the amplitude and frequency equal to those of the main harmonic.

2.3. *Losses in the normal-metal layers and total losses: a non-sinusoidal transport current*

Let us consider the losses caused by a non-sinusoidal transport current $I(t)$ flowing through a coated conductor (in the y-direction) in zero external magnetic field. Losses in the superconductor are given by equation (4). Using the known expressions (see [7, 15, 16]) for the z-component of magnetic field H_z at $z = 0$, corresponding to the increasing and decreasing transport current, respectively,

$$H_z(x, t) = \tilde{H}_z(x, I_{max}, I_c) - \tilde{H}_z(x, I_{max} - I(t), 2I_c) \quad \text{and}$$

$$H_z(x, t) = \tilde{H}_z(x, I_{min}, I_c) + \tilde{H}_z(x, I_{min} - I(t), 2I_c),$$

where

$$\tilde{H}_z(x, I, I_c) = \begin{cases} 0, & |x| < b_i \\ 2H_c \dfrac{x}{|x|} \operatorname{arctanh}\left[\dfrac{x^2 - b_i^2}{a^2 - b_i^2}\right]^{1/2}, & b_i < |x| < a, \end{cases}$$

$$\text{and} \qquad b_i = a\sqrt{1 - I^2/I_c^2}.$$

Substituting H_z into (9), integrating over x, and taking (4) into account, we obtain the normalized total loss power p_i per unit of length of a coated conductor carrying a non-sinusoidal transport current:

$$p_i = \frac{P_{I\text{tot}}}{(\mu_0^2 d_m a/2\rho\pi^3) I_1^2 \omega^2} = F_i + \alpha Q_i(\Delta i), \qquad (11)$$

where $P_{I\text{tot}} = P_I + P_{\text{Ied}}$ and $P_{\text{Ied}} = \frac{\mu_0^2 d_m a I_1^2 \omega^2}{2\rho\pi^2} F_i$ are the total losses and losses in the normal-metal layers per unit length of a coated conductor, respectively,

$$Q_i(\Delta i) = q(\Delta i)/i_1^2, \qquad i_1 = I_1/I_c, \quad i = \frac{I(t)}{I_1},$$

$$F_i = \frac{1}{\omega}\left\{\int_{t_1}^{t_2}\left(\frac{di}{dt}\right)^2 G(b_i)\, dt + \int_{t_2}^{T+t_1}\left(\frac{di}{dt}\right)^2 G\left(\tilde{b}_i\right) dt\right\},$$

$$G(u) = 1 - u - \sqrt{1-u^2}\,\text{Log}\left(\frac{1+\sqrt{1-u^2}}{u}\right) + \frac{1}{2}\left[\text{Log}\left(\frac{1+\sqrt{1-u^2}}{u}\right)\right]^2 \qquad \text{for } u = b_i \text{ or } \tilde{b}_i,$$

$$b_i = \sqrt{1 - \left(\frac{I_{\max} - I(t)}{2I_c}\right)^2},$$

$$\tilde{b}_i = \sqrt{1 - \left(\frac{I_{\min} - I(t)}{2I_c}\right)^2}.$$

Here F_i and αQ_i are the normalized losses in the normal-metal and superconducting strips, respectively.

Expressions (10) and (11) which present losses in a coated conductor in the dimensionless form were obtained for periodic non-sinusoidal external magnetic fields and transport currents varying monotonically between their extreme values.

2.4. Analytical model results

The losses caused by sinusoidal external magnetic fields and transport currents in a coated conductor have been analyzed in [2, 3]. The influence of higher harmonics on losses in a superconducting strip in a non-sinusoidal periodic perpendicular magnetic field has been analyzed in [11]; it was shown that the hysteresis losses per cycle in the Bean critical state model depend only on the difference between the maximum and minimum of the magnetic field $\Delta H = H_{\max} - H_{\min}$, and are independent of the higher harmonic frequencies, see equation (3). The contribution of the higher harmonics to losses is determined by the difference $\Delta H - 2H_1$. Similarly, losses in a superconducting strip with a non-sinusoidal periodic transport current are determined by the difference between the maximum and minimum of the current $\Delta I = I_{\max} - I_{\min}$ (see equation (4)) and contribution of the higher current harmonics—by $\Delta I - 2I_1$. A value of $\Delta H - 2H_1$ ($\Delta I - 2I_1$) is determined by the amplitudes and phases of the higher harmonics. Our analysis shows that contributions of the even and odd harmonics are different. To demonstrate this difference and the importance of taking into account the higher harmonics in AC loss estimates, we shall consider the transport currents of the form $I(t) = I_1 \sin(\omega t) + I_k \sin(k\omega t + \phi_k)$ with $k = 2$ or $k = 3$, and similarly for the external magnetic fields. For the second harmonic ($k = 2$) the maximum of $\Delta I - 2I_1$ is at $\phi_2 = 0$ and can be approximated by $2.4 I_2^{1.8}$; for $k = 3$ this maximum equals $2I_3$ and is at $\phi_3 = \pi$. The maximal ratios for which the current changes monotonically between its extrema are $I_2/I_1 = 0.25$ and $I_3/I_1 = 0.11$.

Losses in the normal-metal layers of coated conductors as well as in a superconducting layer depend on the phase of the higher harmonic. Our calculations have shown that the losses in normal-metal layers are maximal at the same phases ϕ_k at which the losses in a superconductor are: $\phi_2 = 0$ for the second harmonic and $\phi_3 = \pi$ for the third one; below, we analyze these two cases. The situation is similar for the magnetic field problems; see equation (3). The functions Q_h, F_h and Q_i, F_i demonstrate qualitatively the same behavior for $k = 2$ and 3 (see figure 2). The functions Q_i, F_i monotonically increase with the amplitude of the main current harmonic. For $h_1 = H_1/H_c \ll 1$ the functions F_h and Q_h are well fitted by the power laws with exponents 4 and 2, respectively. The function Q_h has a maximum at $h_1 \simeq 2$ and decreases approximately as $1/h_1$ for $h_1 \gg 1$. The function F_h (normalized losses in normal-metal parts of a coated conductor) increases from zero toward a limiting value. At a high external magnetic field, $h_1 \gg 1$, the response of a superconductor can be neglected and losses in normal-metal parts are close to those in the external magnetic field alone. In this case the contribution of the kth harmonic to the normalized loss power F_h in a normal-metal strip in a perpendicular field is $k^2 H_k^2/H_1^2$.

In both considered cases (an external magnetic field and a transport current) the influence of higher harmonics will be characterized by their relative contribution to losses, $\delta P = (P - P_1)/P_1$; here, P is the loss under non-sinusoidal conditions and P_1 denotes the loss caused by the main harmonic. The relative contribution of the third harmonic of the external magnetic field is presented in figure 3. In both the superconductor and the normal metal this contribution is maximal at a low magnetic field (the main harmonic) and decreases as the field increases. Thus, for a low magnetic field $H_1 < H_c$ and $H_3 = 0.05H_1$ the third harmonic contribution to AC losses reaches 20% in the superconductor and 35% in normal-metal parts. For $H_1 \gg H_c$ the contribution decreases to 2% in the normal metal and to 5% in the superconductor (figure 3, red dashed lines).

Let us now consider a coated conductor with transport current (figure 4). For $I_1 \ll I_c$ the losses in the superconductor are well approximated by the expression $P \approx P_1(1 + 4I_3/I_1)$.

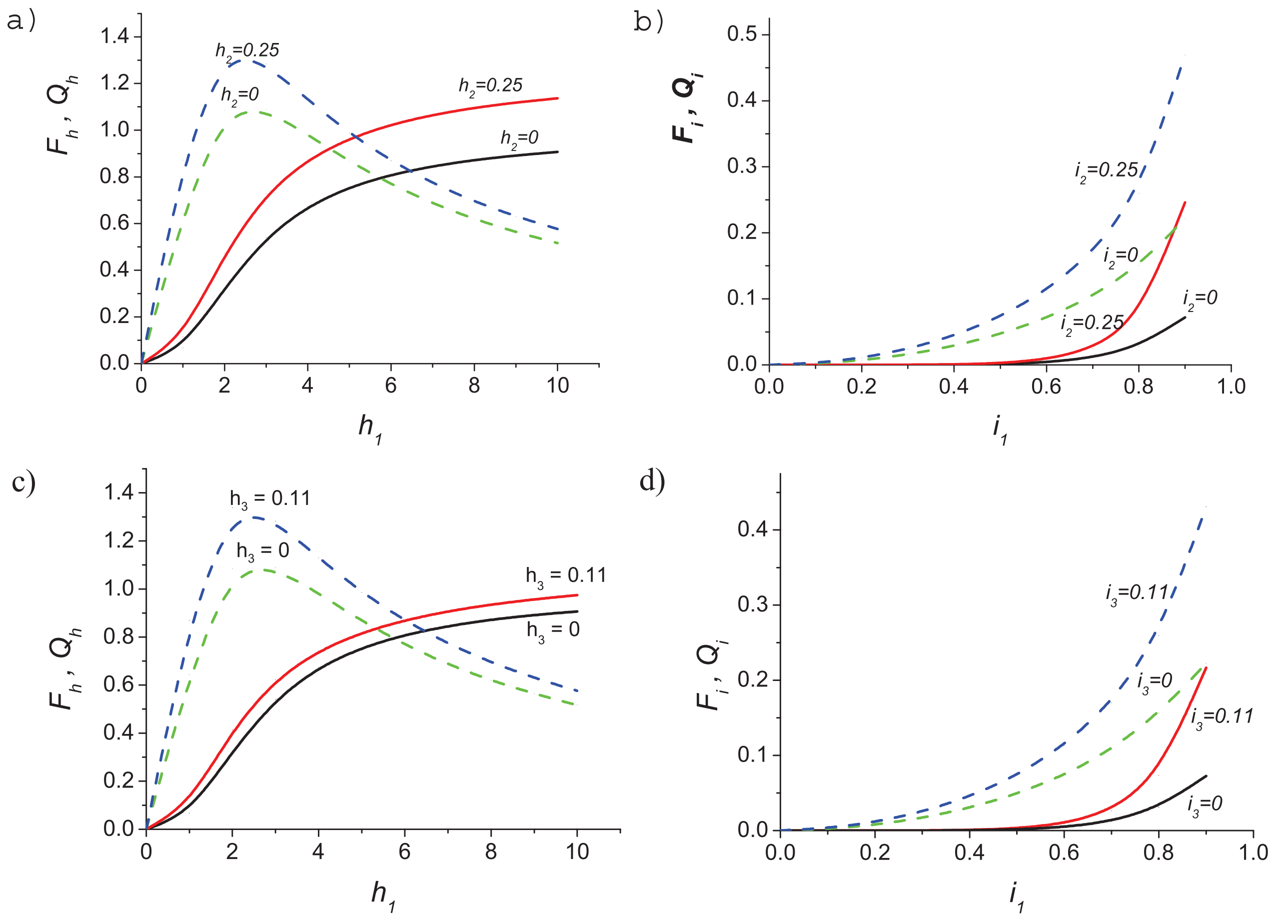

Figure 2. The main harmonic amplitude dependence of Q_h, F_h and Q_i, F_i. (a) Q_h—blue ($h_2 = 0.25$) and green ($h_2 = 0$) dashed lines; F_h—black ($h_2 = 0$) and red ($h_2 = 0.25$) solid lines; $\phi_2 = 0$; (b) Q_i—blue ($i_2 = 0.25$) and green ($i_2 = 0$) dashed lines; F_i—black ($i_2 = 0$) and red ($i_2 = 0.25$) solid lines; $\phi_2 = 0$; (c) Q_h—blue ($h_3 = 0.11$) and green ($h_3 = 0$) dashed lines, F_h—black ($h_3 = 0$) and red ($h_3 = 0.11$) solid lines; $\phi_3 = \pi$; (d) Q_i—blue ($i_3 = 0.11$) and green ($i_3 = 0$) dashed lines, F_i—black ($i_3 = 0$) and red ($i_3 = 0.11$) solid lines; $\phi_3 = \pi$.

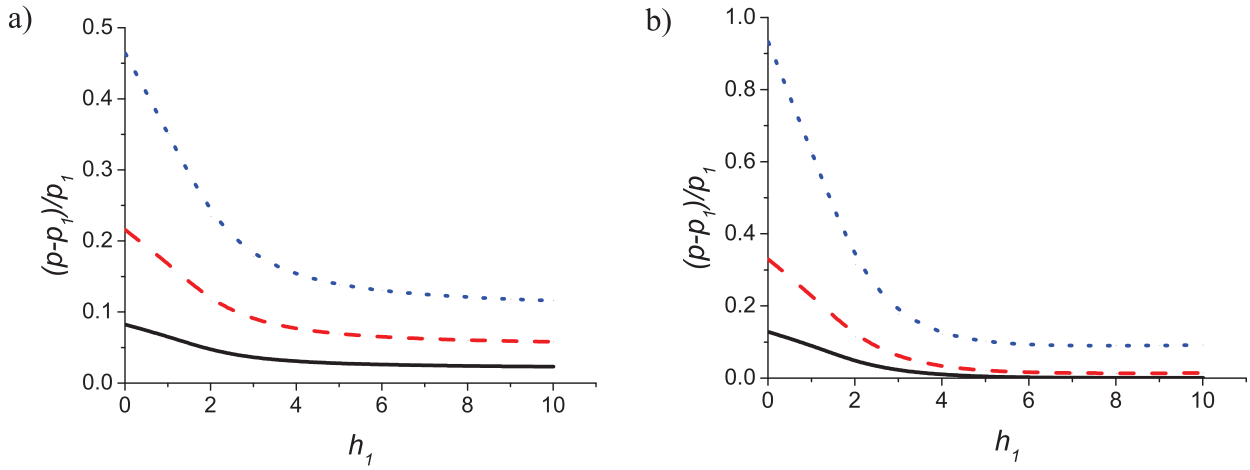

Figure 3. Relative contribution of the third harmonic of the magnetic field to losses in superconductor (a) and normal-metal parts (b). Blue dotted line, $H_3 = 0.1H_1$; red dashed line, $H_3 = 0.05H_1$; black solid line, $H_3 = 0.02H_1$.

However, in contrast to the magnetic field case, the relative contribution of the 5% third harmonic increases with current and, at $I \approx I_c$, is about 45% for the superconductor and 80% for the normal metal (figure 4, red dashed lines).

The relative contributions of losses in the superconducting and normal-metal layers to the total losses depend on the parameter α, which is the same for the current and magnetic field problems (see equations (10) and (11)). The estimation

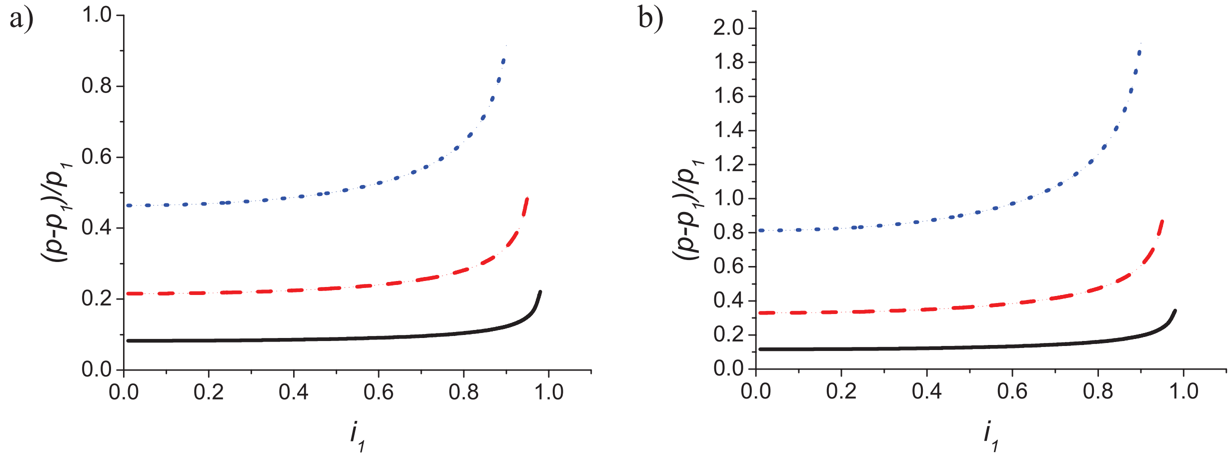

Figure 4. Relative contribution of the third current harmonic to losses in a superconductor (a) and normal-metal parts (b). Blue dotted line, $I_3 = 0.1I_1$; red dashed line, $I_3 = 0.05I_1$; black solid line, $I_3 = 0.02I_1$.

of this parameter and an example of the AC loss calculation are given in the appendix.

3. Numerical analysis of AC losses in coated conductors

We used numerical simulations to calculate the contribution of higher harmonics to AC losses in superconductors with a power law current–voltage characteristic. Here we also assume that the current density in the normal-metal layer is much smaller than that in the superconductor. Therefore, the local E–J relation can be written in the following form:

$$E = \begin{cases} E_0\,(J/J_c)^n & -a_{sc} \le x \le a_{sc}, \\ \rho_0 J & |a_{sc}| < |x| \le |a|, \end{cases} \tag{12}$$

where J is the sheet current density, $E_0 = 1\ \mu\mathrm{V\,cm}^{-1}$ is the electric field caused by the sheet critical current density J_c, $\rho_0 = \rho/d_m$, and $2a$ and $2a_{sc}$ are the widths of the coated conductor (normal-metal strip) and the superconducting layer, respectively. For a coated conductor with surround stabilizer $2a > 2a_{sc}$ [17, 18] and, although the strip ends without the superconductor layer can often be neglected, our mathematical model and numerical scheme have been developed for the general case; in calculations we took $a = 1.05a_{sc}$. The applicability of (12) can be extended by introducing an effective power index as a fitting parameter.

In the infinitely thin strip approximation only the normal component H_z of the AC magnetic field induces losses; in the strip mid-plane, $z = 0$, this component is presented as

$$H_z = -\frac{1}{2\pi}\int_{-a}^{a}\frac{J(u)}{x-u}\,\mathrm{d}u + H_e.$$

Substituting this formula into the equation $\frac{\partial E}{\partial x} = -\mu_0\frac{\partial H_z}{\partial t}$, integrating with respect to x, and setting $E = \tilde{\rho}(x, J)J$ with

$$\tilde{\rho} = \begin{cases} E_0 J^{n-1}/J_c^n & -a_{sc} \le x \le a_{sc} \\ \rho_0 & a_{sc} < |x| \le a, \end{cases}$$

we obtain

$$\tilde{\rho}(x, J)J = \frac{\mu_0}{2\pi}\int_{-a}^{a}\frac{\partial J(u, t)}{\partial t}\ln|x-u|\,\mathrm{d}u - \frac{\partial A_y}{\partial t} + C(t). \tag{13}$$

Here $C(t)$ is an unknown function of time, which appears due to integration through x, determined implicitly by the condition

$$\int_{-a}^{a} J(x, t)\,\mathrm{d}x = I(t), \tag{14}$$

and A_y is the y-component of the vector potential of the external magnetic field. If the applied magnetic field is uniform, the vector potential can be taken as $A_y = H_e x$.

The total loss in a coated conductor per cycle and per unit of length is determined as

$$P = \int_0^T \mathrm{d}t \int_{-a}^{a} EJ\,\mathrm{d}x. \tag{15}$$

The numerical algorithm for solution of equation (13), developed in [19] to analyze the response of a superconductor to a current pulse, was used also in this work to compute AC losses in coated conductors. In our simulations we used a uniform finite element mesh with 400 elements (which ensured the numerical error did not exceed 1%); the time step was chosen adaptively. We assumed zero initial conditions and considered $n = 4$–25. Our calculations showed that, after the first period, the losses per period stabilize and do not change anymore.

In our simulations (figures 5–8) we considered two cases:

- a coated conductor in a non-sinusoidal uniform magnetic field perpendicular to its wide surface without any transport current;
- a coated conductor with a non-sinusoidal transport current in zero magnetic field.

In both cases the relative contribution of a higher harmonic increases with its amplitude (figures 5 and 6). The contribution increases also with the power index n. The results

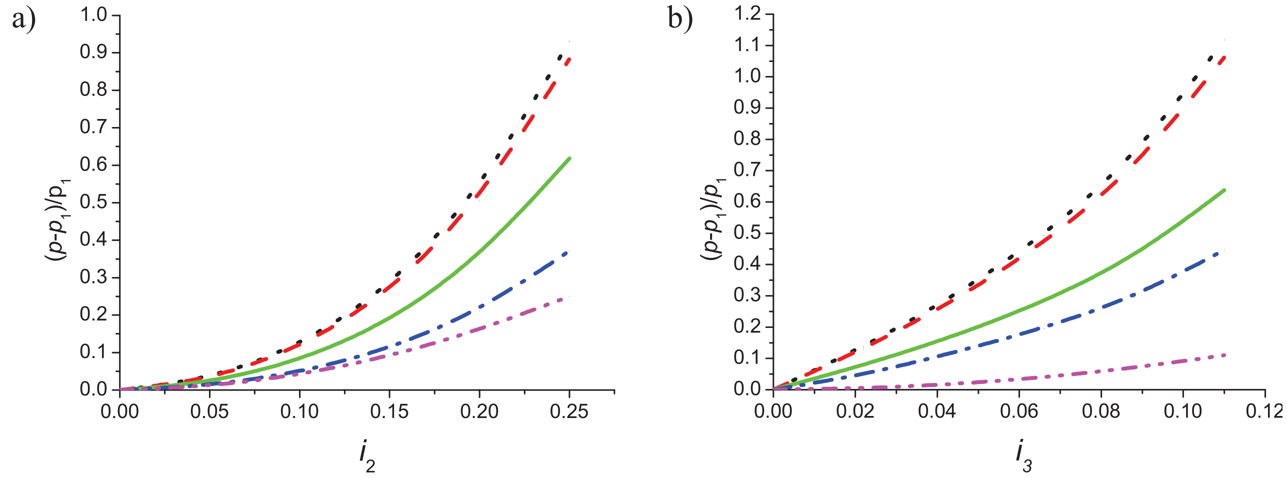

Figure 5. Relative contribution of the higher current harmonic to total losses in a power law coated conductor: (a) the second harmonic, $\phi_2 = 0$; (b) the third harmonic, $\phi_3 = \pi$. Simulation results: the critical state model (black dotted line), the power law model (red dashed, green solid, and blue dash–dotted lines for $n = 25$, 10, and 4, respectively) and normal metal (magenta dash–dot–dotted line) at $i_1 = 0.9$.

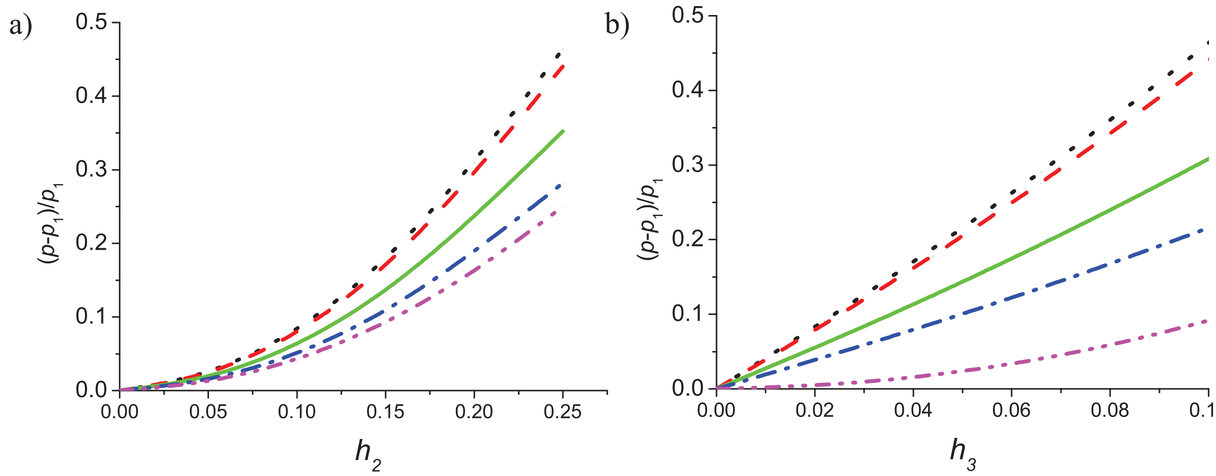

Figure 6. Relative contribution of the higher magnetic field harmonic to total losses in a power law coated conductor: (a) the second harmonic, (b) the third harmonic. Simulation results: the critical state model (black dotted line), the power law model (red dashed, green solid, blue dash-dotted lines for $n = 25$, 10, and 4, respectively) and normal metal (magenta dash–dot-dotted line) at $H_1 = H_c$.

obtained for the power law model with $n = 25$ are very close to those for the critical state model; the higher harmonic contribution to losses is about ten times greater than in normal metal (figure 5(b)). This contribution can reach and even exceed the losses caused by the main harmonic. For $n = 4$ this contribution can reach 45% of that of the main harmonic (figure 5(b)), which is about four times higher than in normal metal (11%).

The relative contribution to losses increases as the main current harmonic grows (figures 7(a) and 8(a)) but decreases as the main field harmonic increases (figures 7(b) and 8(b)).

For a sinusoidal magnetic field the AC losses have been investigated [20] using the high field approximation ($H_1 \gg H_c$) for a field dependent power voltage–current characteristic

$$E = E_0\left(\frac{J}{J_c}\right)^n, \qquad J_c = J_{c0}\frac{H_b}{|H| + H_b}, \tag{16}$$

where J_{c0}, H_b are constants. Extending this approach to strong non-sinusoidal external magnetic fields, the AC losses in a thin superconducting strip have been estimated analytically in [11]:

$$P = P_0\left[\frac{n}{4n+2}\left(\frac{\mu_0 w\omega H_1}{E_0}\right)^{1/n}\right]F, \tag{17}$$

where

$$F = \int_0^T \left\{\frac{\omega|\sum_k \tilde{h}_k k\cos(\omega kt + \phi_k)|^{1+1/n}}{1 + h_b|\sum_k \tilde{h}_k \sin(\omega kt + \phi_k)|}\right\} \mathrm{d}t,$$

$$h_b = H_1/H_b, \qquad \tilde{h}_k = H_k/H_1, \quad (k = 1, 2, \ldots)$$

and $P_0 = 4f\mu_0 J_c a^2 H_1$ denotes the losses given by the critical state model with the magnetic-field-independent critical current in an asymptotically high magnetic field. For a sinusoidal field ($h_k = 0$ for $k > 1$) this gives the estimate [20] and, if $H_b = \infty$ and $n \to \infty$ (the Bean model), we obtain $F \to 4$, so $P \to P_0$. As above, F has the maximum at $\phi_2 = 0$ for the second harmonic and $\phi_3 = \pi$ for the third harmonic.

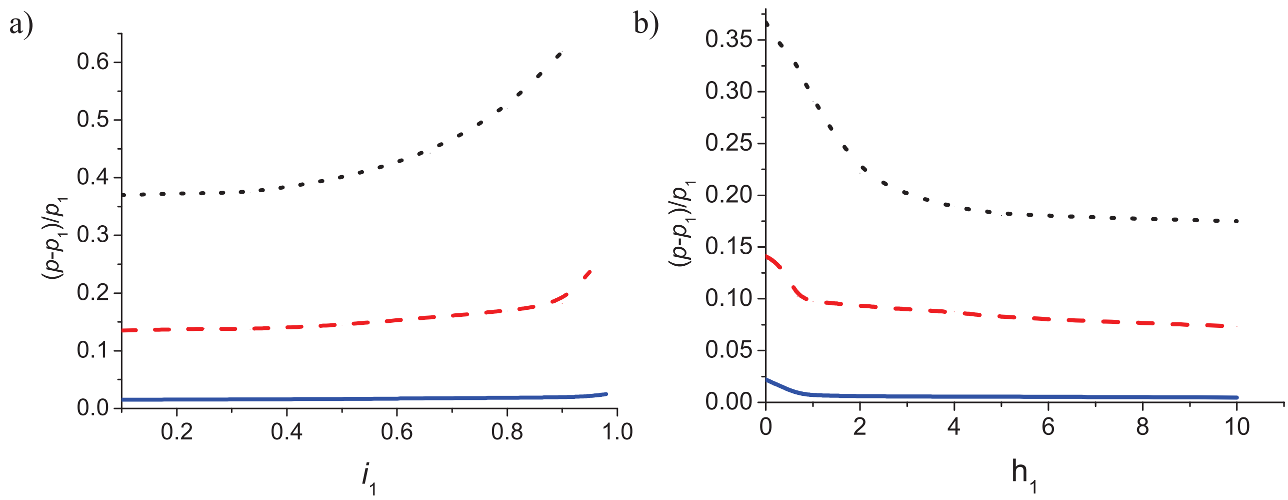

Figure 7. Relative loss contribution of the second current (a) and field (b) harmonic as a function of the main harmonic amplitude. Power law model with $n = 10$, numerical simulations: (a) $i_2 = 0.25i_1$ (black dotted line), $i_2 = 0.15i_1$ (red dashed line), $i_2 = 0.05i_1$ (blue solid line); (b) $h_2 = 0.25h_1$ (black dotted line), $h_2 = 0.15h_1$ (red dashed line), $h_2 = 0.05h_1$ (blue solid line).

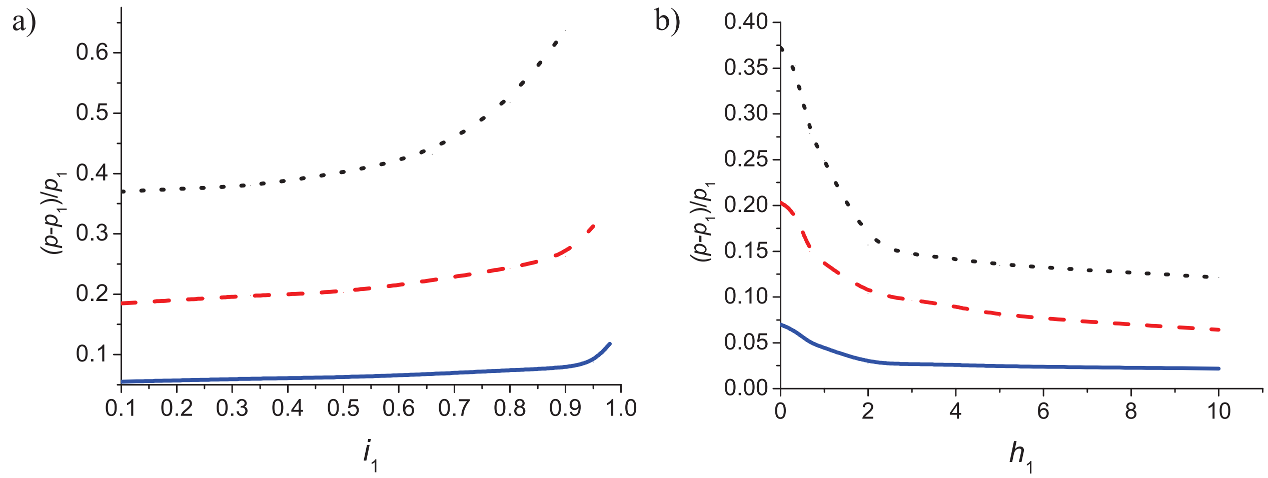

Figure 8. Relative loss contribution of the third current (a) and field (b) harmonic as a function of the main harmonic amplitude. Power law model with $n = 10$, numerical simulations: (a) $i_3 = 0.1i_1$ (black dotted line), $i_3 = 0.05i_1$ (red dashed line), $i_3 = 0.02i_1$ (blue solid line); (b) $h_3 = 0.1h_1$ (black dotted line), $h_3 = 0.05h_1$ (red dashed line), $h_3 = 0.02h_1$ (blue solid line).

The AC loss decreases with the increase of h_b approximately as $1/(1 + 0.85h_b)^{0.625}$ in both cases and increases with the amplitude of the second harmonic as $1 + a_2h_2^{1.8}$ and with the amplitude of the third one as $1 + a_3h_3$ (see [11]). The coefficients a_2 and a_3 depend on the power index n and parameter h_b. For $h_b < 1$ and $n > 10$, within the accuracy of 10%, a_2 and a_3 can be taken as 1.3 and 0.9, respectively. The increase of n above 20 does not lead to any marked further growth (the values of F at $n = 20$ and 40 differ by less than 2%). Comparison of the analytical results with the numerical ones shows that the high magnetic field approximation can be used at least for the main harmonic amplitude higher than $15H_c$.

4. Conclusion

Higher harmonics in coated conductors can substantially change the AC losses, especially in the conductors carrying transport current. In contrast to the normal-metal conductors, in which a 5% third current harmonic causes the loss to increase by about 2% only, the increase due to this harmonic in a coated conductor can reach 50% in the superconducting layer and 90% in the normal-metal parts at $\phi_3 = \pi$ and the amplitude of the main harmonic close to the critical current (see figure 4).

The AC losses obtained by means of numerical simulations using the power law models with high power index ($n \sim 25$) are close to the analytical predictions of the critical state model. The relative loss contributions of higher harmonics of the magnetic field and current increase with the ratio of their amplitude to the amplitude of the main harmonic. Keeping this ratio constant while increasing the main harmonic leads to an increase of the relative contribution in the case of transport current and to a decrease of this contribution in the case of magnetic field. The contribution of a 10% third current harmonic to the total losses (a sum of the losses in the superconductor and normal metal) can reach 110% of the losses caused by the main harmonic; this is about ten times more than the losses caused by the third harmonic in the normal-metal conductor of the same shape (figure 5). Even for a low power index ($n = 4$) the predicted AC losses are substantially higher than the losses in normal metals: the relative contribution of higher harmonics can be four times higher and reach 44% of the losses caused by the main harmonic.

The obtained results show that the losses caused by higher harmonics even of relatively small amplitudes should be taken into account. The analytical expression for AC losses in coated conductors in the high magnetic field approximation is applicable at least for the main harmonic amplitude exceeding $15H_c$.

Appendix

Here we estimate the characteristic values of the parameter α and present two examples of AC loss calculation. According to (10) and (11), the dependence of normalized AC losses on the main harmonic frequency is only via the coefficient $\alpha = \frac{\rho\pi}{\mu_0 a d_m \omega}$, which depends also on the resistivity ρ and cross-section area $2ad_m$ of the normal-metal strip. To estimate this parameter, we consider a coated conductor with the width $2a = 1$ cm and the hastelloy-C substrate thickness of 0.1 mm. The substrate resistivity is 1.24×10^{-6} Ω m, the resistivity of silver and copper layers is about the same and is equal to 2×10^{-9} Ω m at 77 K. Hence, for a coated conductor without protective silver and stabilizer layers [2, 3], the parameter α is about $(10^6\ \mathrm{s}^{-1})/f$; for a conductor with the 2 μm thick silver layer [21], $\alpha \approx (8 \times 10^4\ \mathrm{s}^{-1})/f$; for a well stabilized coated conductor with the copper stabilizer thickness of 100 μm [22], the parameter α is evaluated as $(1.6 \times 10^3\ \mathrm{s}^{-1})/f$. Hence, the losses in a superconductor dominate for the first two conductor types up to high frequencies of the order of 1 MHz. The losses in the normal-metal parts of a well stabilized coated conductor can be comparable or even dominate at frequencies ~1 kHz (these frequencies are used in some special electric power systems, e.g. in airplanes, ships, etc).

The function Q_h decreases as H_c/H_1 at $H_1 \gg H_c$ and at 50–60 Hz the losses in the normal-metal parts of a well stabilized coated conductor dominate in magnetic fields $H_1 > 30H_c$, i.e. when $\mu_0 H_1 > 0.1$ T at the typical value of $J_c = 10^4$ A m^{-1} ($\mu_0 H_c = 0.004$ T). This value is much less than the rated magnetic fields in many power devices. For example, the rated field in a generator is of the order of 1 T, and the losses in the normal-metal parts of the coated conductor operating in such a field dominate. In superconducting power cables the rated fields are of the order of 0.03 T and the total losses are mainly determined by the losses in the superconductor.

The evaluations above were performed for the liquid nitrogen cooling. If coated conductors are cooled by liquid helium, at 4 K, the parameter α decreases at least by an order of magnitude and, for a well stabilized coated conductor, can be estimated as $(1.6 \times 10^2\ \mathrm{s}^{-1})/f$. It is of the order of 1 at the frequency of 50 Hz, so the influence of the current induced in the normal-metal part on the current in the superconductor should be taken into account for accurate loss estimation. In this case our estimate can be considered a crude first approximation. Losses in the superconducting and normal-metal parts of coated conductors become comparable at magnetic fields $H_1 \sim H_c$. At 4 K the typical value of J_c is 1.3×10^5 A m^{-1} and $\mu_0 H_c = 0.05$ T. Thus the losses in the normal-metal parts should be taken into account even for power cables. For example, $\alpha = 4$ for the sinusoidal external magnetic field $H_e = 3H_c \sin(\omega t)$; the losses in the normal metal are about 25% of losses in the superconductor and increase up to 100% when the amplitude of the magnetic field equals 1 T.

As another example, let us estimate the total losses in a well stabilized coated conductor with the critical current $I_c = 100$ A at 77 K; the amplitude I_1 of the main harmonic is $0.9I_c$ and its frequency 400 Hz; the amplitude I_3 of the third harmonic is $0.1I_1$. The norming loss power P_n per unit of the conductor length in equation (11) is $P_n = (\mu_0^2 d_m a/2\rho\pi^3)I_1^2\omega^2 \approx 0.33$ W m^{-1}.

For $I_3 = 0$ we obtain $Q_i = 0.22$ and $F_i = 0.073$ at $I_1 = 0.9I_c$ (figure 2(d)) and the loss powers per unit of the conductor length in the superconductor $P_s = \alpha Q_i P_n = 0.29$ W m^{-1} and in the normal metal $P_m = F_i P_n = 0.024$ W m^{-1}; the total loss power is $P_{I\mathrm{tot}}^{(0\%)} = P_s + P_m = 0.314$ W m^{-1}.

The 10% third current harmonic gives relative contributions $\delta P = 0.9$ for a superconductor (figure 4(a)) and $\delta P = 1.9$ for a normal metal (figure 4(b)). The total loss power per length unit of the coated conductor is $P_{I\mathrm{tot}}^{(10\%)} = 1.9P_s + 2.9P_m \approx 0.62$ W m^{-1}. Thus the 10% third current harmonic increases the losses twice. This harmonic increases the relative contribution of losses in the normal metal from 8% to 13%.

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