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Penetration of spatially nonuniform alternating magnetic field into type-II superconductors

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Abstract

Spatially nonuniform fluctuations of external magnetic field are typically present in applications of superconductors as magnetic bearings and shields. We use the Bean critical-state model to analyze penetration of such alternating fields into type-II superconductors. Simple asymptotic solutions are found for infinite slab configuration and low penetration depth of the magnetic field. To calculate AC losses, we use a vector potential formulation. The solutions obtained are tested by numerical simulations, based on a variational reformulation of the critical-state problem, and by experiments. A BSCCO superconducting bulk specimen with power law voltage–current characteristic was used in experiments.

Keywords: Type-II superconductors; Spatially nonuniform fluctuations; Bean model

To simulate hard superconductor (SC) magnetization and AC losses in a spatially nonuniform alternating external magnetic field, we consider an infinite SC bar with a rectangular cross-section $\Omega = \{-l \leqslant x \leqslant l, -L \leqslant y \leqslant L\}$ placed between two layers of external current (at $x = \pm a$, $a > l$) parallel to z-axis and having the form of a standing wave,

$$I_e = I_0 \sin(2\pi f t) \sin(ky)[\delta(x+a) - \delta(x-a)],$$

where k is the wave number, I_0 is the current amplitude, f is its frequency, and $\delta(x)$ is the delta-function. Inside the SC, the induced current is also directed along the z-axis and does not depend on z. The numerical method we employ is based upon a reformulation of the Bean model as a variational inequality for the current density in the SC (see [1]):

Find $J \in K$ such that for all $t > 0$
$(G * \partial_t\{J + J_e\}, \Phi - J) \geqslant 0$ for any $\Phi \in K$,
and also $J|_{t=0} = J_0(r)$.

Here

$$K = \left\{\Phi(r) : |\Phi| \leqslant J_c \text{ in } \Omega, \int_\Omega \Phi \, d\Omega = 0\right\}$$

is the set of admissible current densities, "$*$" means convolution, $r = (x, y)$, $G(r) = -\ln|r|/2\pi$ is the Green function of the 2d Laplace equation, J_c is the critical current density, $J_0 \in K$ is a given initial distribution of the current density $J(r, t)$ in the cross-section Ω, and $(u, v) = \int u \cdot v \, d\Omega$ is the scalar product of two functions. The convolution $G * J_e$ gives the vector potential of the external field: for $-a < x < a$ we obtain

$$A_e = \mu_0 G * J_e = -\mu_0 I_0/k \sin(2\pi f t) \sin(ky) \exp(-ka) \sinh(kx),$$

where μ_0 is the magnetic permeability of vacuum.

Let us assume that $J_0 = 0$. At $t > 0$, alternating zones of plus and minus critical current densities appear near the SC surface and start to propagate inside. As the

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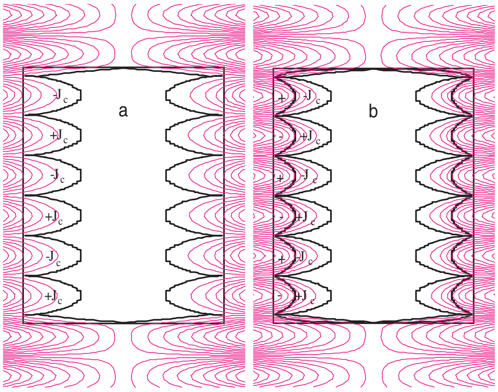

Fig. 1. Penetration of magnetic field fluctuations into SC slab. Thick lines show the boundaries of plus- and minus-critical current zones; thin lines—the magnetic field lines.

external field reaches its maximal strength at $ft = 1/4$ (Fig. 1a) and starts to decrease, zones of the opposite critical current densities appear (Fig. 1b).

For small fluctuations and infinite L, an asymptotic solution can be found [2]: the penetration depth is

$$\Delta_0 = I_0|\sin(ky)|\exp[-k(a-l)]/J_c$$

and the boundary between the zones of the opposite critical current densities may be written as

$$S(y,t) = \Delta_0[1+\sin(2\pi ft)\mathrm{sign}(\cos(2\pi ft))]/2.$$

The approximation is valid up to $\Delta_0 k \approx 0.3$ (see [2]).

To calculate AC losses, one can express the electric field intensity via the total vector and scalar magnetic potentials and exclude the scalar potential by integrating the loss density $p = JE$ over Ω. Periodicity of J in time allows one to exclude also the vector potential of induced current by integrating the loss density in time over one period. The formula for AC losses per period, valid also in the general case, is

$$P = -\int_0^{1/f}\int_\Omega J\partial_t A_e \,\mathrm{d}\Omega\,\mathrm{d}t.$$

Assuming $\Delta_0 k \ll 1$ and using the asymptotic solution, we find the surface density of AC losses analytically:

$$W = \frac{\mu_0 f}{3J_c}\left[J_0 \mathrm{e}^{-k(a-l)}\right]^3(1+\mathrm{e}^{-2kl})|\sin(ky)|^3.$$

In a long wavelength case, $kl \ll 1$, the external field is quasi-uniform, $W \approx 2\mu_0 H_0^3|\sin^3(ky)|/3J_c = W_b|\sin^3(ky)|$, where $H_0 \approx I_0\exp[-k(a-l)]$ is the external field amplitude at the SC surface, W_b is the Bean losses in the uniform field. The case of a short wavelength, $kl \gg 1$, is equivalent to one-sided incidence of the field. The AC losses become $W \approx W_b|\sin^3(ky)|$ but now $H_0 \approx I_0\exp[-k(a-l)]/2$. The loss increase in comparison with the losses in a uniform field is explained by the presence of both, x- and y-field components, and a larger penetration depth, $2H_0/J_c$ instead of H_0/J_c.

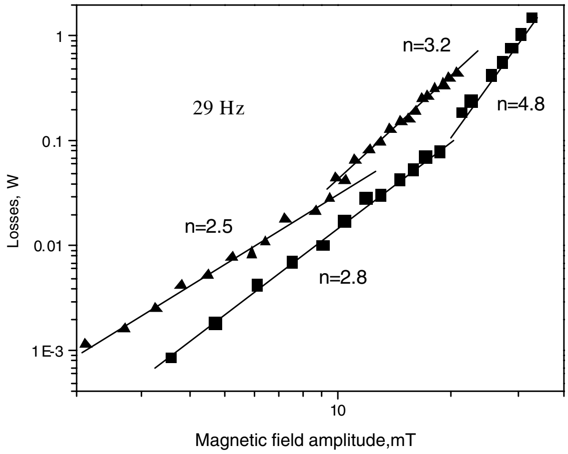

Fig. 2. AC losses in SC cylinder vs. magnetic field assumed to be proportional to the coil current. Squares—losses in a uniform field, triangles—in a nonuniform.

Experimentally, we modelled the short wavelength case using a hollow BSCCO cylinder (outer diameter 70 mm, height 50 mm, wall thickness 2.5 mm) with a power law voltage–current characteristic. A coil, containing 4 sections (outer diameter 60 mm, height 16 mm) with 630 turns each, was placed coaxially inside the cylinder. Connecting the neighbouring sections opposite-in-phase, we obtained, at the SC surface, a sinusoidal-like external field. Its radial and longitudinal component amplitudes were about 13 mT/A and 7 mT/A, respectively. The field exponentially decreased with the radial coordinate.

If the sections are aiding-connected, the field at the SC surface is close to uniform, 34 mT/A, produced by the SC current in the cylinder. The measured AC losses were approximately by a power law, $W = (H_0)^n$ (Fig. 2). Deviation from cubic law for uniform fields was discussed in [3,4]. A likely explanation of the exponent decrease for nonuniform fields is that the penetration depth is not proportional to the field but increases more slowly. The ratio of AC losses in nonuniform and uniform fields is about 2.5; our analytical expression gives 1.7. The difference can result from the finite size of the SC, the deviation of the external field from a sinusoidal one, and a somewhat different current–voltage characteristic.

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