Thin shell model of a coated conductor with a ferromagnetic substrate

I. The model & AC loss   II. Application: HTS dynamo flux pump

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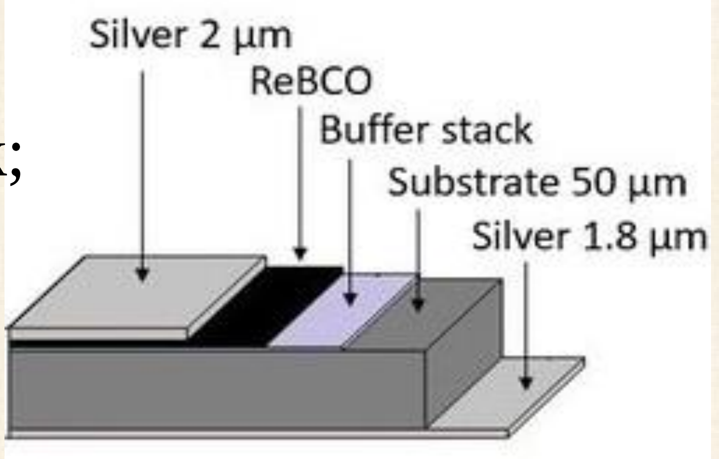
Coated conductor



Width, $2a$: 4-12 mm. Type-II superconducting layer: ≈ 1 μm thick;
The substrate thickness, δ: 30-100 μm.
If the substrate is ferromagnetic, e.g. a Ni-W alloy, it influences the superconducting current density distribution and AC loss.

Modeling approaches

Previous works:

Analytical: Mawatari, 2008 for $\mu_r = \infty$ and the Bean model;

Numerical: Li et al, 2015; Wan et al, 2015; Liu et al, 2017; Hu et al, 2022; Statra et al, 2022; ...

2D formulations and FE solutions; the high width/thickness ratio is a difficulty.

This work: P&S, 2023.

We assume μ_r = const, use this high ratio to employ a thin shell model of the fm substrate, and couple it to the model of an infinitely thin superconducting layer.
Resulting 1D system of integro-differential equations is
(a) much simpler, (b) solved by an accurate and fast Chebyshev spectral method.

Thin shell magnetization model : Krasnov 1977,82,86

Assumptions:

- the thickness of a magnetic shell, δ, is much less than its other sizes;
- the field-independent susceptibility of material $\chi=\mu_r-1\gg 1$.

Asymptotic solution for $\boldsymbol{\delta\to 0}$, $\boldsymbol{\chi\to\infty}$, while $\boldsymbol{\delta\chi}$ remains finite.

The induced field $$\vec{h}^{\,\mathrm{fm}}(r)=-\frac{\chi\delta}{4\pi}\nabla\int_S\vec{\boldsymbol{\sigma}}(s)\cdot\nabla_s\frac{1}{|r-s|}dS,$$

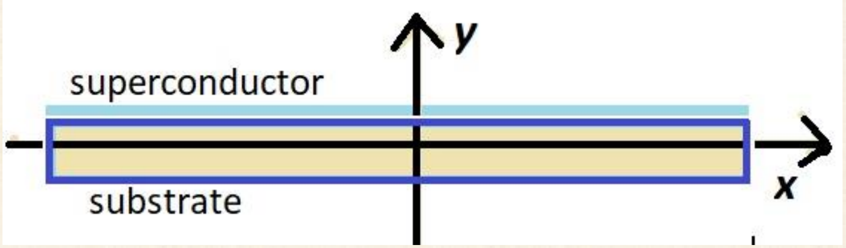
where the "surface magnetization" $\vec{\boldsymbol{\sigma}}$ is tangential to the shell midsurface S and such that $\boldsymbol{\sigma}_n|_{\partial S}=0$ if $\partial S\neq\varnothing$.

The main equation: magnetization in an external field $\vec{h}^{\mathrm{e}}$

$$\vec{\boldsymbol{\sigma}}(s)+\frac{\chi\delta}{4\pi}\nabla_s\int_S\vec{\boldsymbol{\sigma}}(s')\cdot\nabla'_s\frac{1}{|s-s'|}dS'=\chi\delta\vec{h}^{\mathrm{e}}_\tau(s),$$

where $\vec{h}^{\mathrm{e}}_\tau$ is the tangential to S external field component.

Model : thin sc layer + fm substrate



The substrate is a long plate, $\boldsymbol{\sigma}$ is scalar. The equation for $\boldsymbol{\sigma}$ is 1D:

$$(\chi\delta)^{-1}\boldsymbol{\sigma}(t,x)+\frac{\partial}{\partial x}\left(\frac{1}{2\pi}\int_{-a}^{a}\frac{\boldsymbol{\sigma}(t,x')}{x-x'}\mathrm{d}x'\right)=h_x^{\mathrm{e}}(t,x)+h_x^{\mathrm{sc}}(t,x).$$

Here $h_x^{\mathrm{sc}}=\pm j(t,x)/2$ is the sc current-induced field in the substrate.

Superconductor: we assume $e=e_0\left(|j|/j_c\right)^{n-1}j/j_c$ and use the Faraday law

$$\mu_0\frac{\partial}{\partial t}\left(\frac{1}{2\pi}\int_{-a}^{a}\frac{j(t,x')}{x-x'}\mathrm{d}x'+h_y^{\mathrm{e}}(t,x)+h_y^{\mathrm{fm}}(t,x)\right)=\frac{\partial e(t,x)}{\partial x},$$

where $h_y^{\mathrm{fm}}(t,x)=\mp\partial_x\boldsymbol{\sigma}(t,x)/2$ is the substrate-induced field in the sc layer.

There is also the transport curent condition $\int_{-a}^{a} j\mathrm{d}x=I(t).$

Dimensionless formulation

Scaled variables:

$$\tilde{j}=\frac{j}{j_c},\ \tilde{\boldsymbol{\sigma}}=\frac{\boldsymbol{\sigma}}{aj_c},\ \tilde{\boldsymbol{h}}=\frac{\boldsymbol{h}}{j_c},\ \tilde{e}=\frac{e}{e_0},\ (\tilde{x},\tilde{y})=\frac{(x,y)}{a},\ \tilde{t}=\frac{t}{\mu_0 a j_c e_0^{-1}},\ \tilde{I}=\frac{I}{2aj_c}$$

The model (with "~" omitted):

$$\kappa^{-1}\boldsymbol{\sigma}(t,x)+\frac{\partial}{\partial x}\left(\frac{1}{2\pi}\int_{-1}^{1}\frac{\boldsymbol{\sigma}(t,x')}{x-x'}\mathrm{d}x'\right)+\frac{s}{2}j(t,x)=h_x^{\mathrm{e}}(t,x),$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2\pi}\int_{-1}^{1}\frac{j(t,x')}{x-x'}\mathrm{d}x'+h_y^{\mathrm{e}}(t,x)+\frac{s}{2}\frac{\partial\boldsymbol{\sigma}(t,x)}{\partial x}\right)=\frac{\partial e}{\partial x},$$

$$e=|j|^{n-1}j,\qquad \int_{-1}^{1}j\mathrm{d}x=2I(t).$$

Here $\boldsymbol{\kappa}=\chi\delta/a$ is the dimensionless parameter characterizing the substrate, $s=-1$ if the sc layer is above the substrate and $s=1$ if it is below.

Numerical solution (P&S, *IEEE TAS*, **33**, 6601310, 2023) **:**

- Chebyshev spectral discretization in space;
- Method of lines for integration in time.

Advantages :

analytical treatment of singular integrals, fast convergence, simple and accurate matrix representation of linear operations (transition from the mesh values to the interpolating expansions in Chebyshev polynomials and back, differentiation, integration, etc.).

Magnetic field :

$$\boldsymbol{h}(t,x,y)=\boldsymbol{h}^{\mathrm{e}}+\frac{1}{2\pi}\left\{\begin{array}{l} -\displaystyle\int_{-1}^{1}\frac{j(t,x')y+\partial_{x'}\sigma(t,x')(x-x')}{(x-x')^2+y^2}\,\mathrm{d}x' \\ \displaystyle\int_{-1}^{1}\frac{j(t,x')(x-x')-\partial_{x'}\sigma(t,x')y}{(x-x')^2+y^2}\,\mathrm{d}x' \end{array}\right\}$$

Example 1. A transport current problem : convergence

$n = 30, \quad \boldsymbol{h}^{\mathrm{e}} = 0, \quad \kappa = 5, \quad I(t) = 20t$ (dimensionless units).

Numerical solution at the moment when $I = 0.75$ was found for several meshes and compared to the most accurate one.

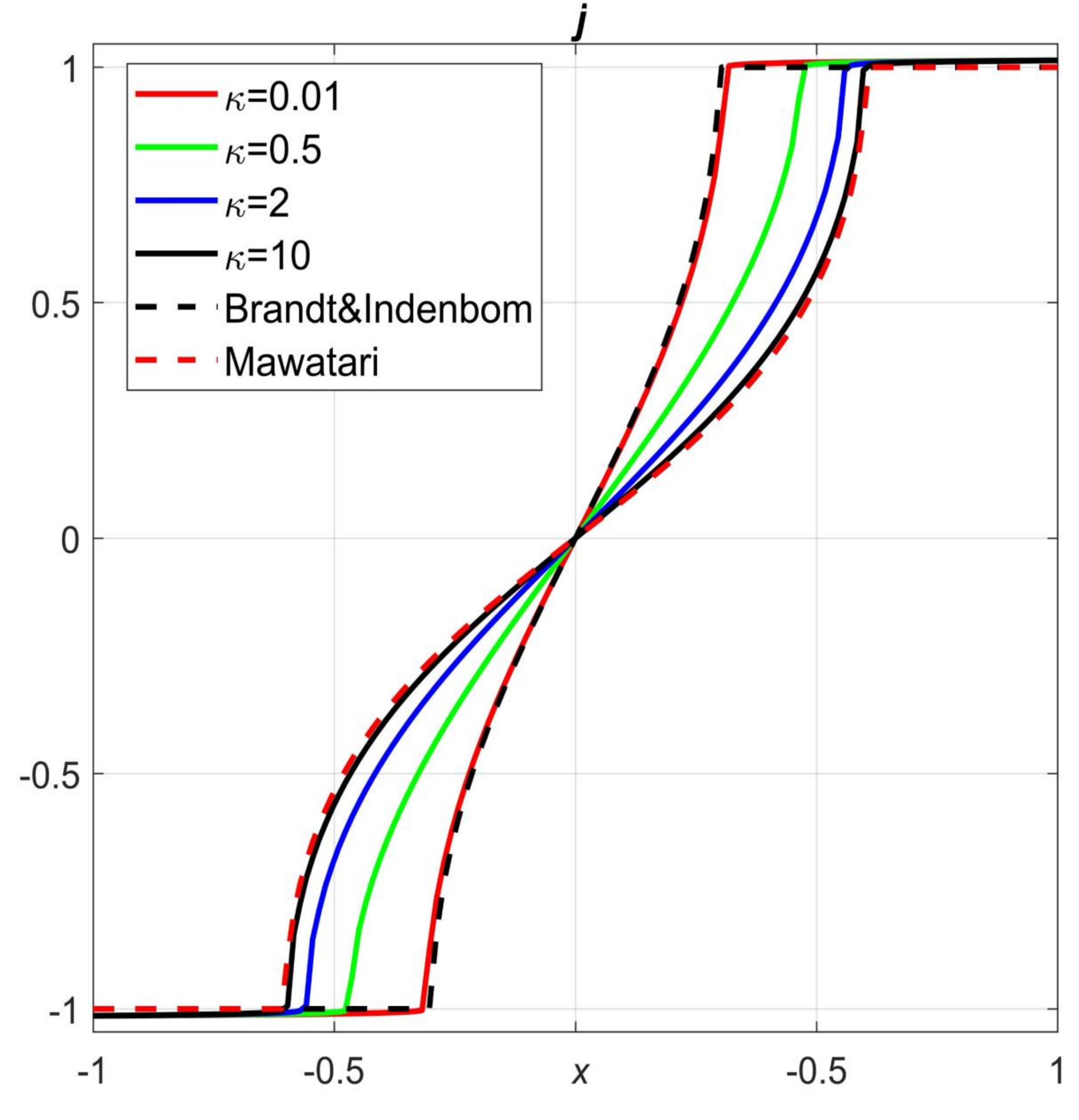
N	$\delta(j)$	$\delta(\sigma)$	CPU time (seconds)
25	9.9e-3	3.8e-3	0.16
50	4.0e-3	1.3e-3	0.17
100	1.4e-3	2.9e-4	0.47
200	1.2e-4	7.5e-5	2.4
400	1.3e-5	1.7e-5	15
800	-	-	112

Here N - the number of mesh points, $\delta(j)$ and $\delta(\sigma)$ - the relative deviations (in the L^1-norm) from the solution with $N = 800$.

Example 2. The sheet current density distribution for different κ

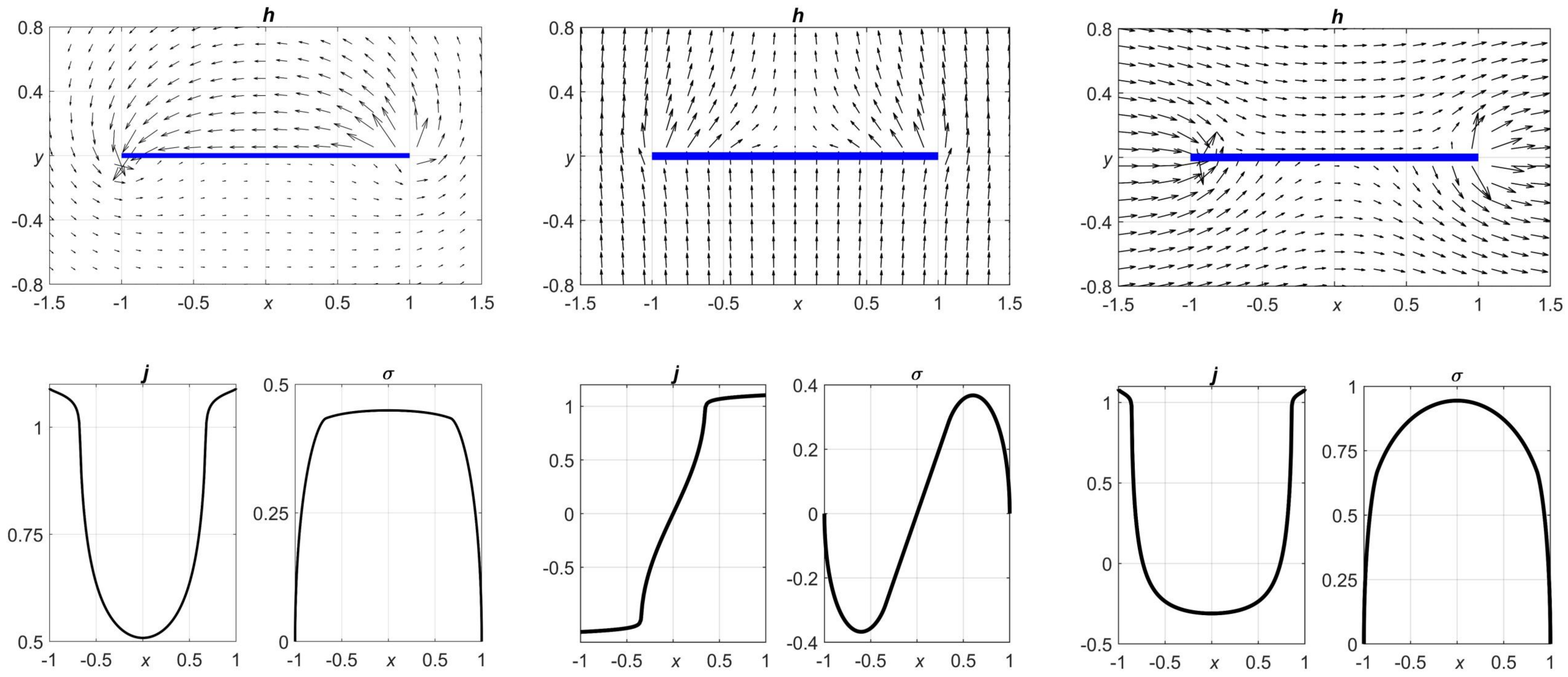
CC in a growing normal field at h_y^e=0.6; here n=200 to compare with the analytical solutions for the Bean model.

For small κ our solution is close to that for a nonmagnetic substrate; for large κ – to the solution for an infinite μ_r



More examples. The sc layer is above the substrate, $\kappa = 5$, $n = 30$.

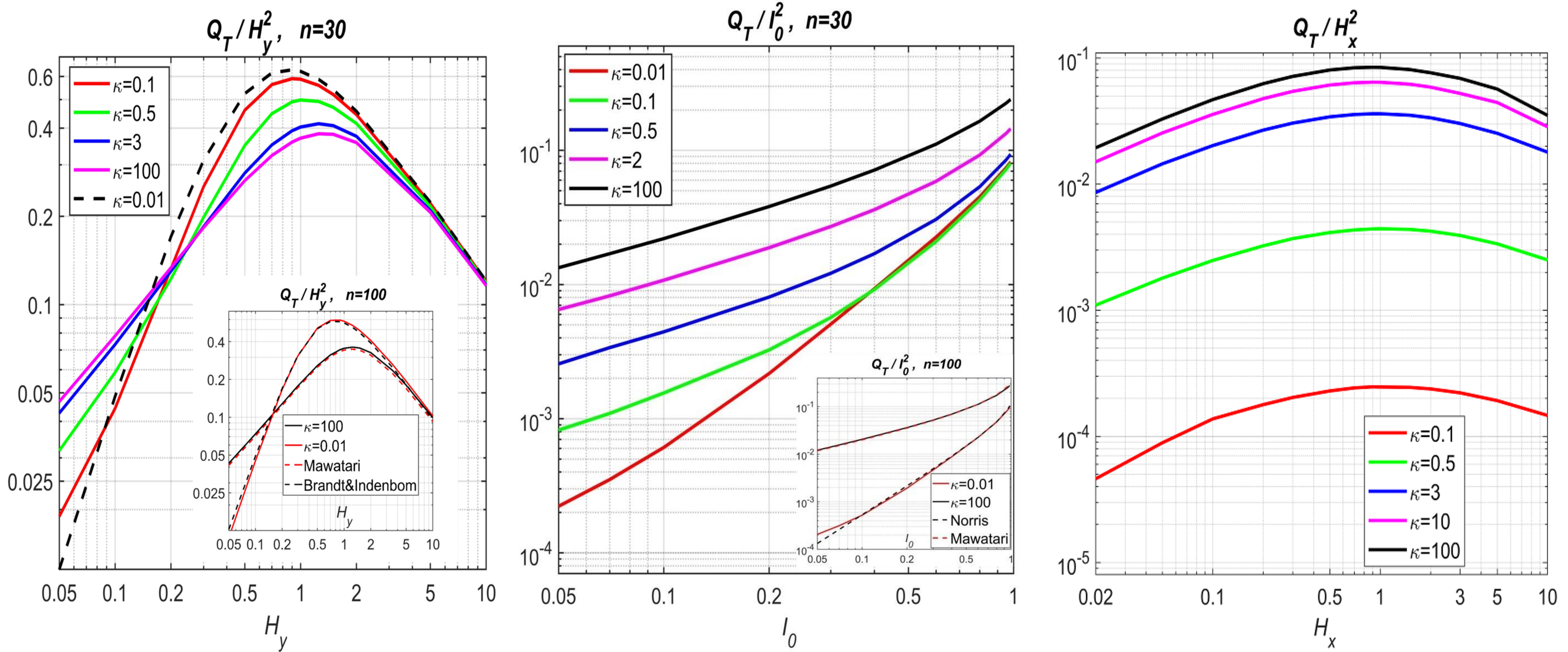
$I = 0.75$ $\qquad$ $h_y^{\mathrm{e}} = 1$ $\qquad$ $h_x^{\mathrm{e}} = 1$



Scaled AC loss per period, dependance on κ and the amplitude of applied field or transport current; n=30, f = 40.

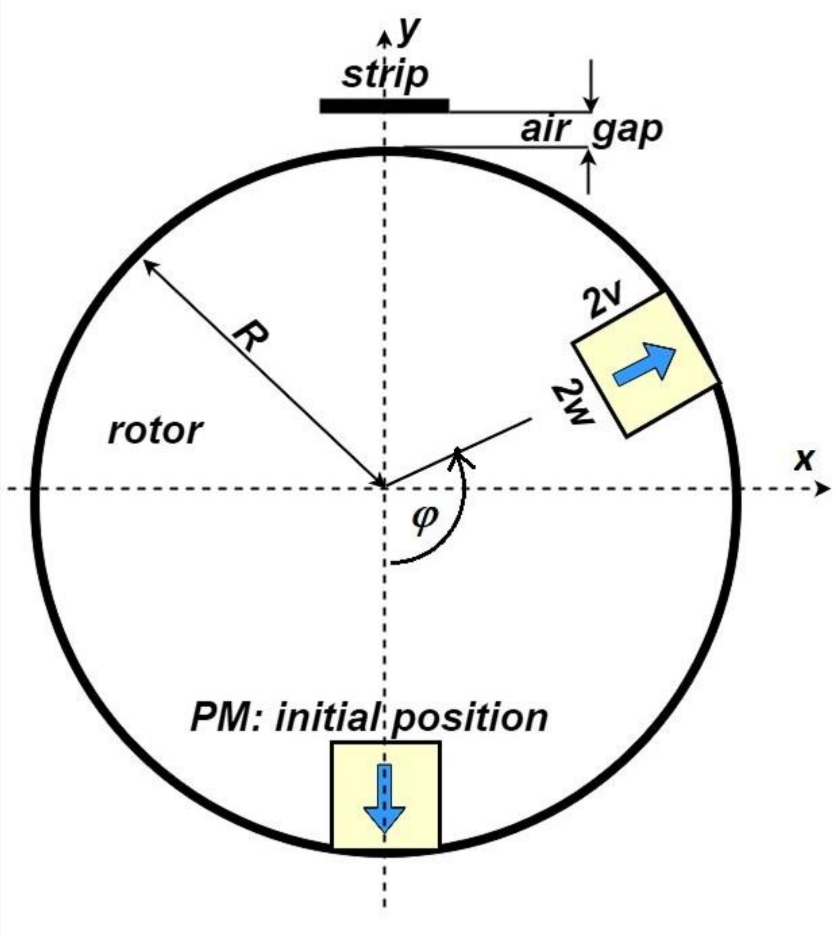
(for a=5 mm, $j_c = 3\cdot 10^4\,\mathrm{Am}^{-1}$ this frequency corresponds to $\approx$ 20Hz).

$$h_y^{\mathrm{e}} = H_y \sin(ft) \qquad I = I_0 \sin(ft) \qquad h_x^{\mathrm{e}} = H_x \sin(ft)$$



HTS dynamo flux pump - contactless charging sc coils or magnets

A simplified model: long permanent magnet + "active length" for voltage,
Mataira et al, 2019; "benchmark problem" Ainslie et al, 2020; P & S, 2021.



To extend this model to the case of a fm stator substrate we use the same thin shell model

$$\kappa^{-1}\sigma(t,x) + \frac{\partial}{\partial x}\left(\frac{1}{2\pi}\int_{-1}^{1}\frac{\sigma(t,x')}{x-x'}\,\mathrm{d}x'\right) + \frac{s}{2}\,j(t,x) = h_x^{\mathrm{e}}(t,x),$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2\pi}\int_{-1}^{1}\frac{j(t,x')}{x-x'}\,\mathrm{d}x' + h_y^{\mathrm{e}}(t,x) + \frac{s}{2}\frac{\partial\sigma(t,x)}{\partial x}\right) = \frac{\partial e}{\partial x},$$

$$e = |j|^{n-1} j, \qquad \int_{-1}^{1} j\,\mathrm{d}x = 2I(t).$$

Now $\boldsymbol{h}^{\mathrm{e}}$ is induced by the rotating permanent magnet (computed, as in P&S 2021, using analytical formulas).

Changing the substrate parameter κ, we study the impact of a fm substrate; all other dynamo parameters are as in Ainslie et al, 2020.

HTS dynamo. The DC voltage.

The instanteneous voltage on the load can be divided into two parts:

- generated in the stator and estimated as

$$V_r(t) = l\langle e\rangle,$$

where $\langle e\rangle = (2a)^{-1}\int_{-a}^{a} e(t,x)\mathrm{d}x$ is the width-averaged e,

l is the effective "active length".

- generated in the closed circuit by the changing magnetic flux Φ.

The second part depends on the circuit configuration. However, Φ is periodic if the transport current is constant and almost periodic in case of charging a coil. Hence, the cycle-average value of $\mathrm{d}\Phi/\mathrm{d}t$ can be neglected. The DC voltage:

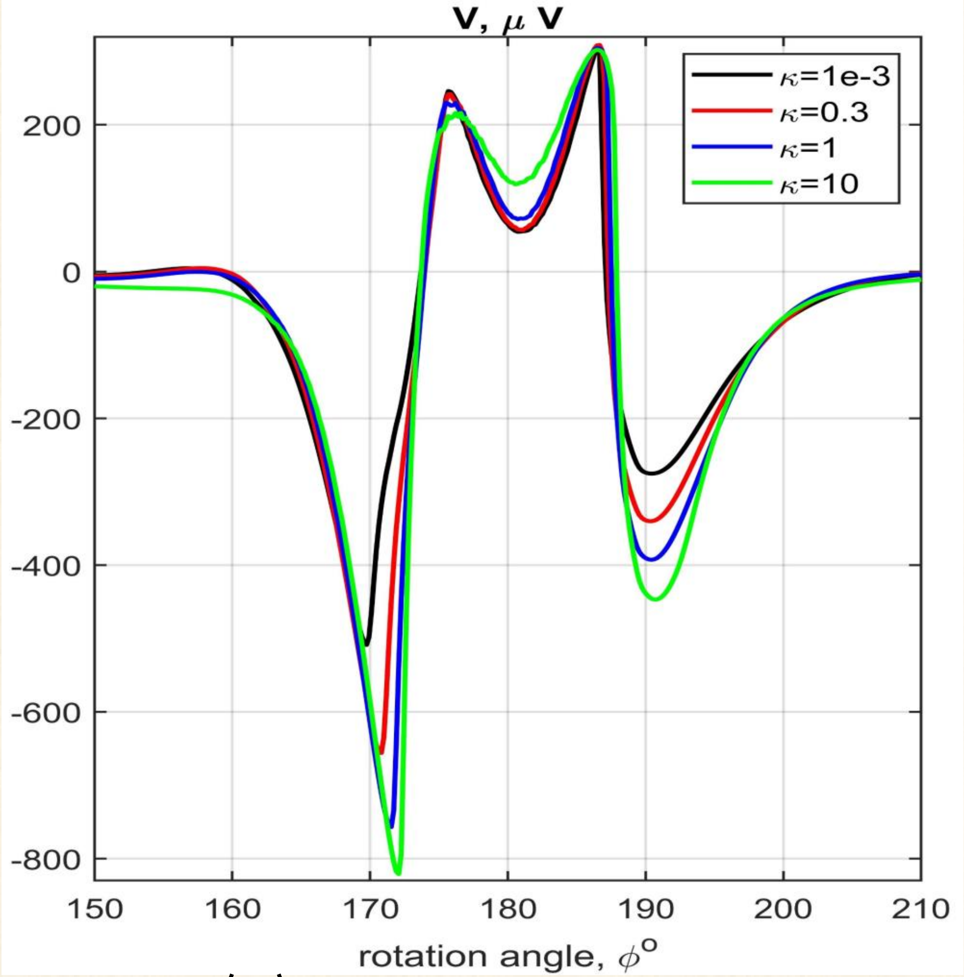
$$\langle V\rangle = f\int_{t_0}^{t_0+1/f} V_r(t)\mathrm{d}t.$$

This is the approach in Mataira et al. 2019, Ainslie et al 2020, P&S 2021,...

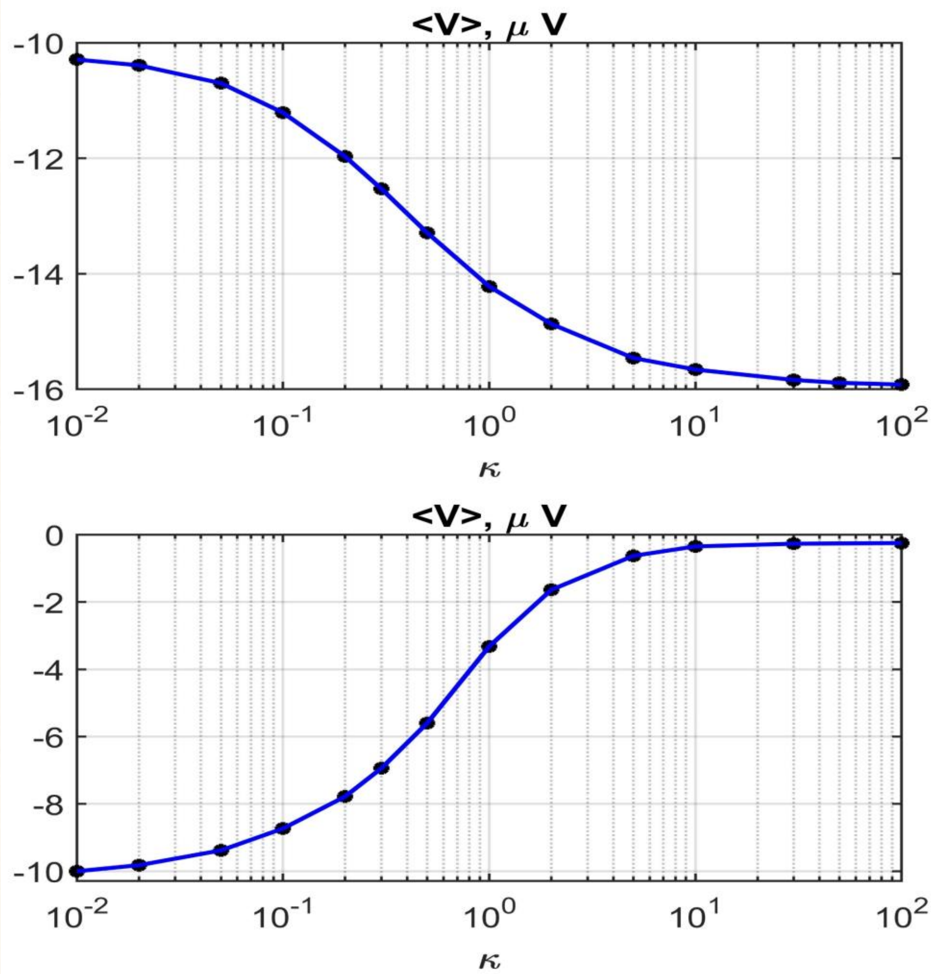
(In several recent works is assumed that the scalar magnetic potential in the Coulomb gauge is the electrostatic potential. This is not true.)

HTS dynamo. Open circuit voltage, simulation results.

If the sc layer is between the rotor and fm substrate, the voltage is increased. In the opposite case the fm substrate shields the sc, the voltage decreases.



$V_r(t) = l\langle e \rangle,$ the sc layer is between the rotor and substrate.



DC voltage. Top: the sc layer is between. Bottom: the sc layer is outside.

HTS dynamo. Charging a coil.

As in Ghabeli et al. 2021, we assume the lumped coil model with $L = 0.24$ mH, $R = 0.88$ μΩ and can supplement the dynamo model by

$$L\frac{\mathrm{d}I}{\mathrm{d}t} + RI = V_r(t, I).$$

Here V_r contains only the $l\langle e\rangle$ part of the instantenous voltage ripples but still has a correct mean-cycle value.

Such simulations are time consuming: charging a coil needs thousands of rotor rotations. A simpler method is based on analysis of problems with a given transport current .

HTS dynamo. A simplified charging model.

- **Problems with a given transport current**

DC voltage dependance on tr. current:

$$\langle V \rangle \approx V_0(\boldsymbol{\kappa}) - R_{\text{eff}}(\boldsymbol{\kappa}) I,$$

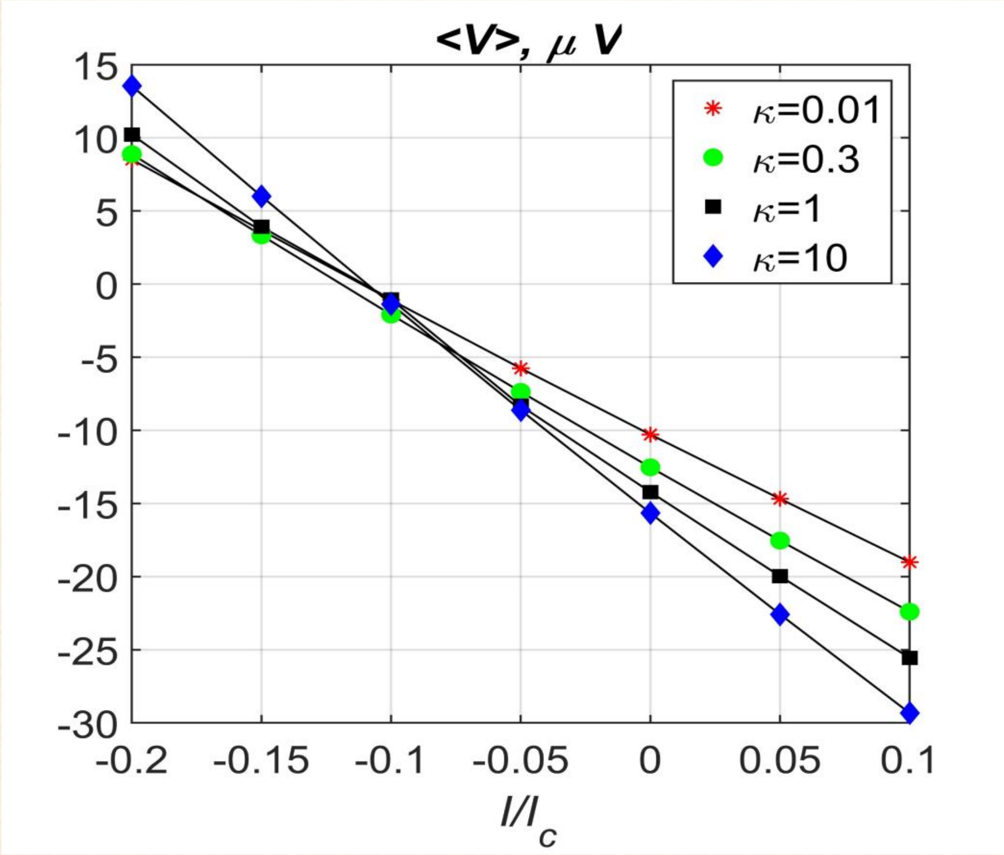
V_0 – the open-circuit DC voltage,

R_{eff} – effective stator resistance.

Let $I_0(\boldsymbol{\kappa})$ be the current for which $\langle V \rangle = 0.$

Then

$$R_{\text{eff}}(\boldsymbol{\kappa}) = V_0(\boldsymbol{\kappa}) / I_0(\boldsymbol{\kappa}).$$



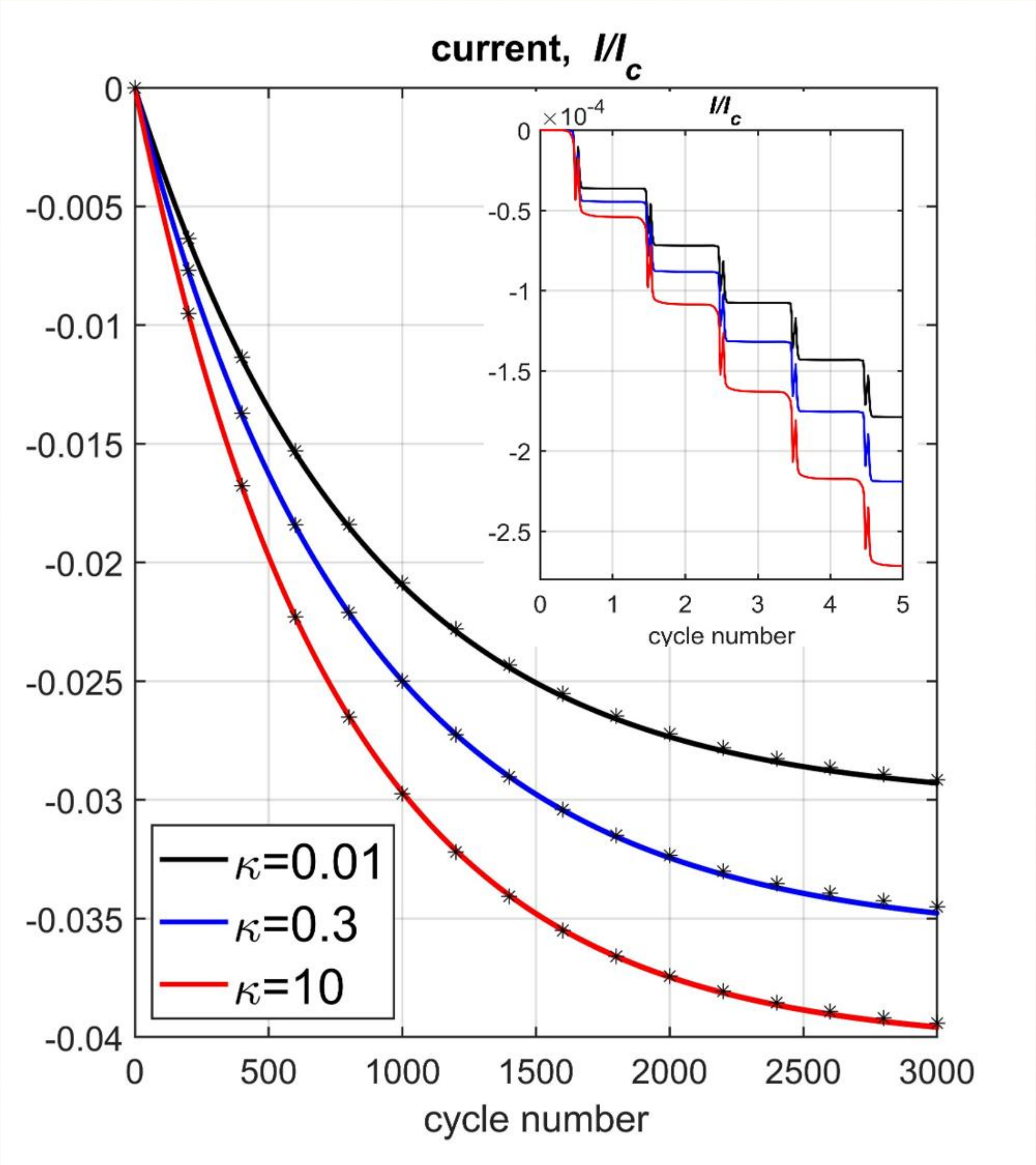
- **Charging model that neglects current ripples**

$$L \frac{\mathrm{d}I}{\mathrm{d}t} + RI = V_0(\boldsymbol{\kappa}) - R_{\text{eff}}(\boldsymbol{\kappa}) I.$$

Solution: $I = I_{\text{sat}}\left[1\text{-exp}(\text{-t}/\boldsymbol{\tau})\right]$, where

$$I_{\text{sat}} = V_0 / (R + R_{\text{eff}}), \quad \boldsymbol{\tau} = L / (R + R_{\text{eff}})$$

HTS dynamo. Comparison of the two charging models.



Solid lines - numerical solution of the thin shell model + circuit eq. for the coil; black stars "*" - analytical solutions for the simplified model.

The two solutions almost coincide. Hence, current ripples (inset) can be ignored when modeling charging a coil.

Conclusion

1. A thin shell model of a coated conductor with a ferromagnetic substrate is derived. It is formulated as a system of 1D integro-differential equations, is much simpler than previous models, and solved by a very efficient spectral method.

2. Using this model, we showed that employing a coated conductor with a fm substrate as a stator can increase the HTS dynamo-generated voltage and accelerate contactless charging a supercon-ducting coil or a magnet. Solving a few transport current problems is needed to find the main parameters of charging without time-consuming simulations.

References: Prigozhin and Sokolovsky, *IEEE TAS* (2023) **33,** 6601310.

Sokolovsky and Prigozhin, *IEEE TAS* (submitted).

Thank you!