

אינטגרל בלתי מסוים

I. חשב את האינטגרלים הבאים תוך שימוש בטבלת האינטגרלים המיידים

- 1) $\int (7e^{-5x} - 6\sqrt{3x} - \pi x^8 + 9\sqrt[3]{x} - 2) dx$ 2. $\int \left(\frac{5}{x} - \frac{7}{\sin^2 3x} + \frac{5}{\cos^2 4x} \right) dx$
- 3) $\int \left(2\cos \frac{x}{3} - \frac{\sin 6x}{4} \right) dx$ 4) $\int (2x-3)^{10} dx$ 5) $\int \frac{dx}{2+3x^2}$ 6) $\int \frac{dx}{\sqrt{2-3x^2}}$
- 7) $\int \frac{dx}{\sqrt{3x^2-2}}$ 8) $\int \frac{dx}{2-3x^2}$ 9) $\int \sin 3x \sin 5x dx$ 10) $\int \cos \frac{x}{2} \cos \frac{x}{3} dx$
- 11) $\int \sin x \cos(x+a) dx$ 12) $\int \cos^2 3x dx$ 13) $\int \left(\frac{1-x}{x} \right)^2 dx$ 14) $\int \frac{x+1}{\sqrt{x}} dx$
- 15) $\int \frac{\sqrt{x}-2\sqrt[3]{x^2}+1}{4\sqrt{x}} dx$ 16) $\int \frac{(\sqrt{2x}-\sqrt[3]{3x})^2}{x} dx$ 17) $\int \frac{\sqrt{x^4+x^{-4}+2}}{x^3} dx$
- 18) $\int \frac{x^2}{1-x^2} dx$ 19) $\int \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1-x^4}} dx$ 20) $\int (2^x + 3^x)^2 dx$
- 21) $\int \frac{2^{x+1} - 5^{x-1}}{10^x} dx$ 22) $\int \frac{e^{3x} + 1}{e^x + 1} dx$ 23) $\int (5 \sinh x - 3 \cosh x) dx$ 24) $\int \operatorname{ctg}^2 x dx$
- 25) $\int \operatorname{tg}^2 x dx$ 26) $\int \sqrt[3]{1-3x} dx$ 27) $\int \frac{dx}{\sqrt{2-5x}}$ 28) $\int \frac{\sqrt[5]{1-2x+x^2}}{1-x} dx$
- 29) $\int (\sin 5x - \sin 5\alpha) dx$ 30) $\int \frac{dx}{\sin^2(2x+\pi/4)}$ 31) $\int \frac{dx}{1+\cos x}$ 32) $\int \frac{dx}{1-\cos x}$
- 33) $\int x(1-x)^{10} dx$ 34) $\int \frac{x^2 dx}{1+x}$ 35) $\int \frac{dx}{\sqrt{x+1} + \sqrt{x-1}}$ 36) $\int x \sqrt{2-5x} dx$
- 37) $\int \ln x dx$ 38) $\int x e^{-x} dx$ 39) $\int x^2 \sin 3x dx$ 40) $\int \arcsin 2x dx$ 41) $\int x \arctan \frac{x}{3} dx$
- 42) $\int e^{3x} \sin 5x dx$ 43) $\int e^{-2x} \cos 3x dx$ 44) $\int \sqrt{5-x^2} dx$ 45) $\int \sqrt{x^2+8} dx$

II. חשב את האינטגרלים הבאים תוך החלפה של משתנה האינטגרציה :

46. a) $\int (2x-3)^{10} dx$ b) $\int \frac{x dx}{\sqrt{1-x^2}}$ c) $\int x^2 \sqrt[3]{1+x^3} dx$ d) $\int \frac{x dx}{3-2x^2}$ e) $\int \frac{x dx}{(1+x^2)^2}$
47. a) $\int \frac{x dx}{4+x^4}$ b) $\int \frac{x^3 dx}{x^8-2}$
- 48.
- a) $\int x e^{-x^2} dx$ b) $\int \left(\sin \frac{1}{x} \right) \frac{dx}{x^2}$ c) $\int \frac{x^2 dx}{\sqrt[3]{(8x^3+27)^2}}$ d) $\int \frac{\ln^2 5x}{x} dx$ e) $\int \frac{dx}{x \ln x \ln(\ln x)}$
- f) $\int \frac{dx}{\cos x}$ g) $\int \frac{\arctan x}{1+x^2} dx$ h) $\int \frac{dx}{\arcsin^2 x \sqrt{1-x^2}}$ i) $\int \frac{dx}{1+\sin x}$ j) $\int \frac{dx}{1+\cos x}$

$$49. a) \int \frac{1+x}{\sqrt{1-x^2}} dx \quad b) \int \frac{3x-1}{x^2+9} dx \quad c) \int \frac{(8x-11)dx}{\sqrt{5+2x-x^2}} \quad d) \int \frac{(x+2)dx}{x^2+2x+2}$$

$$e) \int \frac{(3x-1)dx}{\sqrt{x^2+2x+2}} \quad f) \int \frac{(x-2)dx}{x^2-7x+12}$$

III. אינטגרציה של פונקציות טריגונומטריות :

$$50 a) \int \sin^5 2x \cos 2x dx \quad b) \int \tan 5x dx \quad c) \int \cot 3x dx \quad d) \int \frac{\sin x dx}{\sqrt{\cos^3 x}} \quad e) \int \frac{dx}{\sin x}$$

$$51. a) \int \sin 3x \sin 5x dx \quad b) \int \cos \frac{x}{2} \cos \frac{x}{3} dx \quad c) \int \sin x \cos(x+a) dx \quad d) \int \cos^2 3x dx$$

$$e) \int \sin^3 5x dx \quad f) \int \cos^4 2x dx \quad g) \int \tan^3 7x dx$$

IV. אינטגרציה של פונקציות ראציונליות :

$$52. a) \int \frac{(2x+3)dx}{(x-2)(x+5)} \quad b) \int \frac{x^3 dx}{x^2+x-2} \quad c) \int \frac{(x^3+1)dx}{x^3-5x^2+6x} \quad d) \int \frac{(x^2+1)dx}{(x+1)^2(x-1)} \quad e) \int \frac{xdx}{x^3-1}$$

אינטגרציה של פונקציות אי-רציונליות :

$$53. \int \frac{1}{\sqrt{x-1}} dx \quad 54. \int \frac{2}{4+\sqrt{x-2}} dx \quad 55. \int \frac{1}{x+\sqrt[3]{x}} dx \quad 56. \int \frac{\sqrt[3]{x}-1}{\sqrt{x+1}} dx$$

חשב את האינטגרלים הבאים באמצעות הצבות טריגונומטריות

$$57. \int \frac{dx}{(x^2+9)^{3/2}} \quad 58. \int \frac{dx}{(1-x^2)^{3/2}} \quad 59. \int \frac{dx}{(x^2+4)^2}$$

V. תרגילי אינטגרציה שונים

$$1. \int \frac{dx}{e^x + e^{-x}} \quad 2. \int \frac{dx}{(\arcsin x)^2 \sqrt{1-x^2}} \quad 3. \int \frac{\cos x dx}{\sqrt{2+\cos 2x}} \quad 4. \int \frac{xdx}{x^4+3x^2+2}$$

$$5. \int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} dx \quad 6. \int \frac{dx}{\sin^2 x \cos^2 x} \quad 7. \int \frac{\sin x \cos^3 x}{1+\cos^2 x} dx$$

$$8. \int \frac{\arctg \sqrt{x}}{\sqrt{x}} \cdot \frac{dx}{1+x} \quad 9. \int \frac{1-\sqrt{x+1}}{1+\sqrt[3]{x+1}} dx \quad 10. \int \sqrt{2+x+x^2} dx$$

$$11. \int x^3 e^{-x^2} dx \quad 12. \int \cos^6 x dx \quad 13. \int \frac{\arctg e^x}{e^x} dx \quad 14. \int \frac{dx}{x^3+1}$$

$$15. \int \frac{dx}{3x^2-2x-1} \quad 16. \int x \sin^2 x dx \quad 17. \int \sqrt{2+x-x^2} dx \quad 18. \int \frac{\sin x \cos^3 x}{9+\cos^8 x} dx$$

$$19. \int \frac{\cos x - \sin x + 1}{\cos x + \sin x + 1} dx \quad 20. \int \frac{dx}{\cos x(3+\sin x)} \quad 21. \int \frac{dx}{x(x^3+1)} \quad 22. \int \frac{\ln x}{(2x+5)^3} dx$$

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$$13) \left(\frac{1-x}{x}\right)^2 = \frac{1}{x^2} - \frac{2}{x} + 1 \quad 17) x^4 + x^{-4} + 2 = (x^2 + x^{-2})^2$$

$$18) \frac{x^2}{1-x^2} = \frac{x^2-1+1}{1-x^2} = -1 + \frac{1}{1-x^2} \quad 19) \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1-x^4}} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1+x^2}}$$

$$20) (2^x + 3^x)^2 = 2^{2x} + 2 \cdot 6^x + 3^{2x}$$

$$21) \frac{2^{x+1} - 5^{x-1}}{10^x} = 2\left(\frac{1}{5}\right)^x - \frac{1}{5}\left(\frac{1}{2}\right)^x \quad 22) \frac{e^{3x} + 1}{e^x + 1} = \frac{(e^x + 1)(e^{2x} - e^x + 1)}{e^x + 1} = e^{2x} - e^x + 1$$

$$24) \operatorname{ctg}^2 x = -1 + \frac{1}{\sin^2 x} \quad 25) \operatorname{tg}^2 x = -1 + \frac{1}{\cos^2 x} \quad 28) \sqrt[5]{1-2x+x^2} = (1-x)^{2/5}$$

$$31) 1 + \cos x = 2\cos^2(x/2) \quad 32) 1 - \cos x = 2\sin^2(x/2)$$

$$33) x(1-x)^{10} = -(-1+1-x)(1-x)^{10} = -(1-x)^{11} + (1-x)^{10}$$

$$34) \frac{x^2-1+1}{1+x} = x-1 + \frac{1}{1+x} \quad 35) \frac{1}{\sqrt{x+1} + \sqrt{x-1}} = \frac{\sqrt{x+1} - \sqrt{x-1}}{(x+1) - (x-1)}$$

$$36) x\sqrt{2-5x} = \frac{1}{5}(2-2+5x)\sqrt{2-5x} = \frac{2}{5}\sqrt{2-5x} - \frac{1}{5}(2-5x)^{3/2}$$

$$46.c) \int x^2 \sqrt[3]{1+x^3} dx = \left| \begin{array}{l} 1+x^3 = t \\ 3x^2 dx = dt \end{array} \right| = \frac{1}{3} \int \sqrt[3]{t} dt$$

$$d) \int \frac{x dx}{3-2x^2} \quad 3-2x^2 = t \quad e) \int \frac{x dx}{(1+x^2)^2} \quad 1+x^2 = t$$

$$47.a) \int \frac{x dx}{4+x^4} = \frac{1}{2} \int \frac{dx^2}{4+x^4}, \quad x^2 = t, \quad b) \frac{x^3 dx}{x^8-2} = \frac{1}{4} \frac{dx^4}{(x^4)^2-2}$$

$$48.a) x dx = -\frac{1}{2} d(-x^2) \quad b) \frac{dx}{x^2} = -d\left(\frac{1}{x}\right) \quad c) x^2 dx = \frac{1}{2} d(x^3) \quad d) \frac{dx}{x} = d(\ln 5x)$$

$$e) \frac{dx}{x \ln x \ln(\ln x)} = \frac{d(\ln x)}{\ln x \ln(\ln x)} = \left| \ln x = t \right| = \frac{dt}{t \ln(t)} = \frac{d(\ln t)}{\ln t} = \frac{du}{u}$$

$$49.a) \int \frac{x dx}{\sqrt{1-x^2}} = -\frac{1}{2} \int \frac{d(1-x^2)}{\sqrt{1-x^2}} = -\frac{1}{2} \int \frac{du}{\sqrt{u}}$$

$$\int \frac{1+x}{\sqrt{1-x^2}} dx = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x dx}{\sqrt{1-x^2}}$$

$$c) \int \frac{(8x-11)dx}{\sqrt{5+2x-x^2}} = \left| \begin{array}{l} 5+2x-x^2 = -(x^2-2x-5) = \\ = -[(x-1)^2-1-5] = 6-(x-1)^2 \\ x-1=t, dx=dt \end{array} \right| = \int \frac{(8(t+1)-11)dt}{\sqrt{6-t^2}} =$$

$$\int \frac{8t dt}{\sqrt{6-t^2}} - 3 \int \frac{dt}{\sqrt{6-t^2}}$$

$$57. \int \frac{dx}{(x^2+9)^{3/2}} = \left| \begin{array}{l} x=3\tan t, \quad dx=3dt/\cos^2 t \\ x^2+9=9(\tan^2 t+1)=9/\cos^2 t \\ (x^2+9)^{3/2}=27/\cos^3 t \end{array} \right| = \int \frac{\cos^3 t}{27} \frac{3dt}{\cos^2 t} =$$

$$\frac{1}{9} \int \cos t dt = \frac{1}{9} \sin t + C = \frac{1}{9} \frac{x}{\sqrt{x^2+9}} + C$$

$$58. \int \frac{dx}{(1-x^2)^{3/2}} = \left| \begin{array}{l} x=\sin t, \quad dx=\cos t dt \\ 1-x^2=\cos^2 t \end{array} \right| = \int \frac{\cos t dt}{\cos^3 t} = \tan t + C = \frac{x}{\sqrt{1-x^2}} + C$$

$$59. \int \frac{dx}{(x^2+4)^2} = \left| \begin{array}{l} x=2\tan t, \quad dx=2dt/\cos^2 t \\ x^2+4=4(\tan^2 t+1)=4/\cos^2 t \\ (x^2+4)^2=16/\cos^4 t \end{array} \right| = \int \frac{\cos^4 t}{16} \frac{2dt}{\cos^2 t} = \frac{1}{8} \int \cos^2 t dt =$$

$$\frac{1}{8} \int \frac{1+\cos 2t}{2} dt = \frac{1}{16} t + \frac{1}{32} \sin 2t + C = \left| \begin{array}{l} x=2\tan t \Rightarrow t = \arctan \frac{x}{2} \\ \sin 2t = 2 \sin t \cos t = 2 \frac{x}{\sqrt{x^2+4}} \frac{2}{\sqrt{x^2+4}} \end{array} \right| =$$

$$\frac{1}{16} \arctan \frac{x}{2} + \frac{1}{8} \frac{x}{x^2+4} + C$$

תרגילי אינטגרציה שונים

$$1. \int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x dx}{e^{2x} + 1} = \int \frac{d(e^x)}{e^{2x} + 1} = \arctan e^x + C$$

$$3. \int \frac{\cos x dx}{\sqrt{2+\cos 2x}} = \frac{1}{\sqrt{2}} \int \frac{d(\sqrt{2} \sin x)}{\sqrt{3-2\sin^2 x}} = \frac{1}{\sqrt{2}} \arcsin \left(\sqrt{\frac{2}{3}} \sin x \right) + C$$

$$5. \int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} dx \quad a^2 \sin^2 x + b^2 \cos^2 x = t$$

$$18) \int \frac{\sin x \cos^3 x}{9 + \cos^8 x} dx = -\int \frac{\cos^3 x}{9 + \cos^8 x} d(\cos x) = -\int \frac{z^3}{9 + z^8} dz = -\frac{1}{4} \int \frac{dz^4}{9 + z^8} = -\frac{1}{4} \int \frac{dt}{9 + t^2} =$$

$$-\frac{1}{12} \arctan \frac{\cos^4 x}{3} + C$$

$$19) \int \frac{\cos x - \sin x + 1}{\cos x + \sin x + 1} dx = \int \frac{2\cos^2 \frac{x}{2} - 2\sin \frac{x}{2} \cos \frac{x}{2}}{2\cos^2 \frac{x}{2} + 2\sin \frac{x}{2} \cos \frac{x}{2}} dx = \int \frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}} dx =$$

$$= \int \frac{2d\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)}{\cos \frac{x}{2} + \sin \frac{x}{2}} = 2 \ln \left| \cos \frac{x}{2} + \sin \frac{x}{2} \right| + C = \ln |1 + \sin x| + C$$