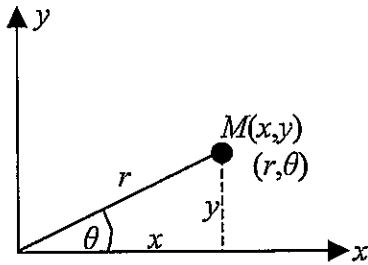


### I. מספרים מרוכבים



מספר מדומה:  $i^2 = -1$ ,  $i$

מספר מרוכב:  $x, y \in \mathbb{R}$ ,  $z = x + yi$

הפעולות בין שני מספרים מרוכבים:

$$z_1 = -1 + 3i, z_2 = 2 + 5i$$

$$z_1 + z_2 = (-1+2) + (3+5)i = 1 + 8i, \quad z_1 - z_2 = (-1-2) + (3-5)i = -3 - 2i,$$

$$z_1 \cdot z_2 = (-1+3i) \cdot (2+5i) = -2 + 6i - 5i - 15 = -17 + i,$$

$$\frac{z_1}{z_2} = \frac{-1+3i}{2+5i} = \frac{-1+3i}{2+5i} \cdot \frac{2-5i}{2-5i} = \frac{13+11i}{4+25} = \frac{13}{29} + \frac{11}{29}i$$

הצגה הטריגונומטרית:

$$z = x + yi = r(\cos\theta + i\sin\theta), \quad r = |z| = \sqrt{x^2 + y^2}, \quad \tan\theta = y/x$$

דוגמאות:

$$1 - i = \sqrt{2} \left( \cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right) \right), \quad -1 + i\sqrt{3} = 2 \left( \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} \right)$$

נוסחאות:

$$r_1(\cos\theta_1 + i\sin\theta_1) \cdot r_2(\cos\theta_2 + i\sin\theta_2) = r_1 \cdot r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$$

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$$

$$(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta), \quad n \in \mathbb{N}$$

$$\sqrt[n]{z} = \sqrt[n]{r(\cos\theta + i\sin\theta)} = \sqrt[n]{r} \left( \cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n} \right), \quad k = 0, 1, 2, \dots, n-1$$

דוגמאות:

$$1. (-1 + i\sqrt{3})^3 = \left( 2 \left( \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} \right) \right)^3 = 2^3 \left( \cos\left(\frac{2\pi}{3} \cdot 3\right) + i\sin\left(\frac{2\pi}{3} \cdot 3\right) \right) = 8$$

$$2. z^4 + 16 = 0 \Rightarrow z^4 = -16 = 16(\cos\pi + i\sin\pi)$$

$$\sqrt[4]{-16} = 2 \left( \cos\frac{\pi + 2\pi k}{4} + i\sin\frac{\pi + 2\pi k}{4} \right), \quad k = 0, 1, 2, 3$$

$$z_0 = 2 \left( \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} \right) = \sqrt{2} + i\sqrt{2}$$

$$z_1 = 2 \left( \cos\frac{\pi + 2\pi}{4} + i\sin\frac{\pi + 2\pi}{4} \right) = -\sqrt{2} + i\sqrt{2}$$

$$z_2 = 2 \left( \cos\frac{\pi + 4\pi}{4} + i\sin\frac{\pi + 4\pi}{4} \right) = -\sqrt{2} - i\sqrt{2}$$

$$z_3 = 2 \left( \cos\frac{\pi + 6\pi}{4} + i\sin\frac{\pi + 6\pi}{4} \right) = \sqrt{2} - i\sqrt{2}$$

### תרגילים

#### I. מספרים מרוכבים

1.  $z_1 = 3 - 2i$  ,  $z_2 = 2 + 4i$

חשב : א)  $z_1 - z_2, z_1 + z_2$  ב)  $z_1^3, z_1^2, z_1 \cdot z_2$  ג)  $\frac{z_1}{z_2}$

2. חשב :  $i^{10}, i^{83}, i^{97}, i^{4n}, i^{4n+1}, i^{4n+2}, i^{4n+3}$

3. פתור את המשוואות :  $z^2 - 2z + 2 = 0$  ,  $z^2 + 2z + 5 = 0$

4. חשב בעזרת נוסחת מואבר :  $(-1+i)^5$  ,  $(\sqrt{3}+i)^8$

5. מצא את כל הפתרונות של המשוואות הבאות :

1)  $z^3 - 1 = 0$  2)  $z^2 + 16i = 0$  3)  $z^3 + 8 = 0$  4)  $z^4 - 1 = 0$

#### II. אינטגרלים

1.  $\int \left( 2 \cos \frac{x}{3} - \frac{\sin 6x}{4} + 7e^{-5x} - 6\sqrt{3x} + \frac{5}{x} - \pi x^8 + 9\sqrt[3]{x} - \frac{7}{\sin^2 3x} + \frac{5}{\cos^2 4x} - 2 \right) dx$

2. a)  $\int (2x-3)^{10} dx$  b)  $\int \frac{x dx}{\sqrt{1-x^2}}$  c)  $\int x^2 \sqrt[3]{1+x^3} dx$  d)  $\int \frac{x dx}{3-2x^2}$  e)  $\int \frac{x dx}{(1+x^2)^2}$

3. a)  $\int \frac{dx}{2+3x^2}$  b)  $\int \frac{dx}{\sqrt{2-3x^2}}$  c)  $\int \frac{dx}{\sqrt{3x^2-2}}$  d)  $\int \frac{dx}{2-3x^2}$  e)  $\int \frac{x dx}{4+x^4}$  f)  $\int \frac{x^3 dx}{x^8-2}$

4. a)  $\int x e^{-x^2} dx$  b)  $\int \left( \sin \frac{1}{x} \right) \frac{dx}{x^2}$  c)  $\int \frac{x^2 dx}{\sqrt[3]{(8x^3+27)^2}}$  d)  $\int \frac{\ln^2 5x}{x} dx$  e)  $\int \frac{dx}{x \ln x \ln(\ln x)}$

f)  $\int \sin^5 2x \cos 2x dx$  g)  $\int \tan 5x dx$  h)  $\int \cot 3x dx$  i)  $\int \frac{\sin x dx}{\sqrt{\cos^3 x}}$  j)  $\int \frac{dx}{\sin x}$

k)  $\int \frac{dx}{\cos x}$  l)  $\int \frac{\arctan x}{1+x^2} dx$  m)  $\int \frac{dx}{\arcsin^2 x \sqrt{1-x^2}}$  n)  $\int \frac{dx}{1+\sin x}$  o)  $\int \frac{dx}{1+\cos x}$

5. a)  $\int \frac{1+x}{\sqrt{1-x^2}} dx$  b)  $\int \frac{3x-1}{x^2+9} dx$  c)  $\int \frac{(8x-11) dx}{\sqrt{5+2x-x^2}}$  d)  $\int \frac{(x+2) dx}{x^2+2x+2}$

e)  $\int \frac{(3x-1) dx}{\sqrt{x^2+2x+2}}$  f)  $\int \frac{(x-2) dx}{x^2-7x+12}$

6. a)  $\int \sin 3x \sin 5x dx$  b)  $\int \cos \frac{x}{2} \cos \frac{x}{3} dx$  c)  $\int \sin x \cos(x+a) dx$  d)  $\int \cos^2 3x dx$

e)  $\int \sin^3 5x dx$  f)  $\int \cos^4 2x dx$  g)  $\int \tan^3 7x dx$

7. a)  $\int \frac{(2x+3) dx}{(x-2)(x+5)}$  b)  $\int \frac{x^3 dx}{x^2+x-2}$  c)  $\int \frac{(x^3+1) dx}{x^3-5x^2+6x}$  d)  $\int \frac{(x^2+1) dx}{(x+1)^2(x-1)}$  e)  $\int \frac{x dx}{x^3-1}$

8. a)  $\int \ln x dx$  b)  $\int x e^{-x} dx$  c)  $\int x^2 \sin 3x dx$  d)  $\int \arcsin 2x dx$  e)  $\int x \arctan \frac{x}{3} dx$

9. a)  $\int e^{3x} \sin 5x dx$  b)  $\int e^{-2x} \cos 3x dx$  c)  $\int \sqrt{5-x^2} dx$  d)  $\int \sqrt{x^2+8} dx$

תשובות :

I. מספרים מרוכבים

1. א.  $1-6i, 5+2i$  (ב.  $-9-46i, 5-12i, 14+8i$ )  $\lambda$   $-0.1-0.8i$  .1  
 2.  $-i, -1, i, 1, i, -i, -1$  .2  
 3.  $-1 \pm 2i, 1 \pm i$  .3  
 4.  $4-4i, -128-128\sqrt{3}i$  .4  
 5.  $\pm 1, \pm i$  (4  $-2, 1 \pm \sqrt{3}i$ ) (3  $2\sqrt{2}(1-i), 2\sqrt{2}(-1+i)$ ) (2  $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ ) (1 .5

II. אינטגרלים

1.  $6 \sin \frac{x}{3} + \frac{\cos 6x}{24} - \frac{7}{5} e^{-5x} - 4x\sqrt{3x} + 5 \ln|x| - \frac{\pi x^9}{9} + \frac{27}{4} \sqrt[3]{x^4} + \frac{7}{3} \cot 3x + \frac{5}{4} \tan 4x - 2x + C$

2. a)  $\frac{1}{22}(2x-3)^{11} + C$  b)  $C - \sqrt{1-x^2}$  c)  $0.25 \sqrt[3]{(1+x^3)^4} + C$

d)  $C - 0.25 \ln|3-2x^2|$  e)  $C - \frac{0.5}{1+x^2}$

3. a)  $\frac{\sqrt{6}}{6} \arctan \frac{x\sqrt{6}}{2} + C$  b)  $\frac{\sqrt{3}}{3} \arcsin \frac{x\sqrt{6}}{2} + C$  c)  $\frac{\sqrt{3}}{3} \ln|\sqrt{3}x + \sqrt{3x^2-2}| + C$

d)  $\frac{\sqrt{6}}{12} \ln \left| \frac{x\sqrt{3} + \sqrt{2}}{x\sqrt{3} - \sqrt{2}} \right| + C$  e)  $0.25 \arctan \frac{x^2}{2} + C$  f)  $\frac{\sqrt{2}}{16} \ln \left| \frac{x^4 - \sqrt{2}}{x^4 + \sqrt{2}} \right| + C$

4. a)  $C - \frac{1}{2} e^{-x^2}$  b)  $\cos \frac{1}{x} + C$  c)  $\frac{1}{8} \sqrt[3]{8x^3+27} + C$  d)  $\frac{\ln^3 5x}{3} + C$  e)  $\ln|\ln(\ln x)| + C$

f)  $\frac{1}{12} \sin^6 2x + C$  g)  $C - \frac{1}{5} \ln|\cos 5x|$  h)  $C + \frac{1}{3} \ln|\sin 3x|$  i)  $\frac{2}{\sqrt{\cos x}} + C$

j)  $\frac{1}{2} \ln \left| \frac{1-\cos x}{1+\cos x} \right| + C$  or  $\ln \left| \tan \frac{x}{2} \right| + C$  k)  $\frac{1}{2} \ln \left| \frac{1+\sin x}{1-\sin x} \right| + C$  or  $\ln \left| \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \right| + C$

l)  $\frac{1}{2} \arctan^2 x + C$  m)  $C - \frac{1}{\arcsin x}$  n)  $C - \tan \left( \frac{\pi}{4} - \frac{x}{2} \right)$  o)  $C + \tan \frac{x}{2}$

5. a)  $\arcsin x - \sqrt{1-x^2} + C$  b)  $\frac{3}{2} \ln(x^2+9) - \frac{1}{3} \arctan \frac{x}{3} + C$

c)  $C - 8\sqrt{5+2x-x^2} - 3 \arcsin \frac{x-1}{\sqrt{6}}$  d)  $0.5 \ln(x^2+2x+2) + \arctan(x+1) + C$

e)  $3\sqrt{x^2+2x+2} - 4 \ln|x+1+\sqrt{x^2+2x+2}| + C$  f)  $\ln \frac{(x-4)^2}{|x-3|} + C$

6. a)  $\frac{1}{4} \sin 2x - \frac{1}{16} \sin 8x + C$  b)  $3 \sin \frac{x}{6} + \frac{3}{5} \sin \frac{5x}{6} + C$  c)  $C - \frac{1}{4} \cos(2x+a) - \frac{\sin a}{2} x$

d)  $\frac{1}{12} \sin 6x + \frac{1}{2} x + C$  e)  $C - \frac{1}{5} \cos 5x + \frac{1}{15} \cos^3 5x$

f)  $\frac{1}{8} \sin 4x + \frac{1}{64} \sin 8x + \frac{3}{8} x + C$  g)  $\frac{1}{14} \tan^2 7x + \frac{1}{7} \ln|\cos 7x| + C$

$$7. a) \ln |(x-2)(x+5)| + C \quad b) C - x + \frac{1}{2}x^2 + \frac{8}{3} \ln |x+2| + \frac{1}{3} \ln |x-1|$$

$$c) x + \frac{1}{6} \ln |x| - \frac{9}{2} \ln |x-2| + \frac{28}{3} \ln |x-3| + C \quad d) \frac{1}{x+1} + \frac{1}{2} \ln |x^2 - 1| + C$$

$$e) \frac{1}{3} \ln |x-1| - \frac{1}{6} \ln (x^2 + x + 1) + \frac{1}{\sqrt{3}} \arctan \frac{2x+1}{\sqrt{3}} + C$$

$$8. a) C - x(1 - \ln x) \quad b) C - (x+1)e^{-x} \quad c) C - \frac{1}{3}x^2 \cos 3x + \frac{2}{27} \cos 3x + \frac{2}{9}x \sin 3x$$

$$d) x \arcsin 2x + \frac{1}{2} \sqrt{1-4x^2} + C \quad e) C - \frac{3}{2}x + \frac{9+x^2}{2} \arctan \frac{x}{3}$$

$$9. a) \frac{3 \sin 5x - 5 \cos 5x}{34} e^{3x} + C \quad b) \frac{3 \sin 3x - 2 \cos 3x}{13} e^{-2x} + C$$

$$c) \frac{x}{2} \sqrt{5-x^2} + \frac{5}{2} \arcsin \frac{x}{\sqrt{5}} + C \quad d) \frac{x}{2} \sqrt{x^2+8} + 4 \ln \left| x + \sqrt{x^2+8} \right| + C$$