OPERATOR THEORY AND OPERATOR ALGEBRAS

Organizers: Daniel Alpay and Orr Shalit

Ben-Gurion University of the Negev, Beer-Sheva

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ABSTRACTS

TBA,

Daniel Alpay, Department of Mathematics, BGU

We start the study of Schur analysis in the quaternionic setting using the theory of slice hyperholomorphic functions. The novelty of our approach is that slice hyperholomorphic functions allows to write realizations in terms of a suitable resolvent, the so called S-resolvent operator and to extend several results that hold in the complex case to the quaternionic case. We discuss reproducing kernels, positive definite functions in this setting and we show how they can be obtained in our setting using the extension operator and the slice regular product. We define Schur multipliers, and find their co-isometric realization in terms of the associated de Branges-Rovnyak space. We define Blaschke factors and Blaschke products and we consider a related interpolation problem. We will also discuss the case of Pontryagin spaces. This is joint work with Fabrizio Colombo and Irene Sabadini (Politechnico Milano)

On the curvature inequality

Shibananda Biswas, Department of Mathematics, BGU

The curvature of a contraction in the Cowen-Douglas class of rank one on the unit disc is bounded above by the curvature of the backward shift operator. However, in general, an operator satisfying the curvature inequality need not be contractive. We find a stronger inequality for the curvature which forces contractivity of the operator. We describe a generalization to the case of commuting tuples. This is a joint work with Dinesh Keshari and Gadadhar Misra.

Weighted Bergman spaces: shift-invariant subspaces and input/state/output linear systems

Vladimir Bolotnikov, College of William and Mary

It is well known that backward shift invariant subspaces of the Hardy space over the unit disk occur as the images of observability operators associated with a discrete-time linear system with stable state-dynamics, while forward shift-invariant subspaces have a representation in terms of an inner function. We will discuss these issues in the Bergman space context. This is a joint work with J. A. Ball.

Trace formulas for Toeplitz-like operators

Harry Dym, Weizmann Institute of Science

In this talk I will give an expository account of some trace formulas that arose when I was trying to understand continuous analogues of the strong Szegö formulas and variations thereof many years ago. The latter part of the talk will be based on recent joint work with David Kimsey.

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Extension operators in Geometric Function Theory

Mark Elin, Ort Braude College

Since the work of Roper and Suffridge in 1995, there has been considerable interest in constructing holomorphic mappings of the unit ball in a Banach space with various geometric properties by using mappings with similar properties acting in a subspace. Such properties include convexity, starlikeness, spirallikeness, and so on. It is also of interest to extend subordination chains, semigroups and semigroup generators.
The main purpose of the talk is to demonstrate new methods and some recent results. In particular, we show that the extension of each spirallike mapping is A-spirallike for a variety of linear operators A. We combine dynamic and geometric approaches. In particular, we use new one-dimensional covering results which are of independent interest.

The stable rank of subalgebras of a nest algebra

Abraham Feintuch, Department of Mathematics, BGU

The algebra of causal stable linear time varying systems is a nest algebra and the question: *is every stabilizable linear system strongly stabilizable* is equivalent to the question: *is the stable rank of the nest algebra equal to one*. The fundamental result of S. Treil gives that this is the case for linear time-invariant systems and I have shown that this is not the case for continuous time. The discrete time case has not been solved. We discuss subalgebras of the nest algebra which arise naturally in the theory of linear systems. Some of them properly contain H^∞ and others don't.

Controllability and observability of networks of linear systems

Paul Fuhrmann, Department of Mathematics, BGU

The object of the talk is the better understanding of controllability and observability properties of heterogeneous networks of linear systems. It extends prior work by Hara et al. who characterized controllability for homogeneous networks of identical linear SISO systems. Our approach is based on extending the classical notion of strict system equivalence to networks of linear systems. We survey and extend known characterizations for controllability and observability for arbitrary interconnected linear MIMO systems. Both static and dynamic interconnection laws are considered and various applications to classes of homogeneous and heterogeneous networks are derived. This is joint work with Uwe Helmke.

Decomposable approximations of nuclear C*-algebras

Ilan Hirshberg, Department of Mathematics, BGU

TBA

Analytic, algebraic, and group theoretic, tools in wavelets and filters

Palle E. T. Jorgensen, The University of Iowa, Iowa City, IA

The talk will begin with multiresolutions, and then turn to matrix valued functions of one or more complex variables, motivated by signal processing, the lifting schemes, and lifting algorithms. Multibands suggest higher order matrix functions which offer their own challenges.
Sample result: Under suitable restrictions, in the case of polynomial entries, these matrix functions factor into finite products of alternating upper and lower diagonal matrix functions. Even though pioneering ideas are from engineering, we hope to show that they are of interest in pure mathematics as well, especially in operator theory.
The results are also of practical significance in designs for building filters.
One of our motivations here is the desire to extend and refine existing methods (for the case of two bands) to the case of multiple bands. In the simplest case, by this we mean that signals are viewed as time function (discrete time) and each time-function generating a frequency

response function (generating function) of a complex variable. In many applications it is possible to encode time-signals or their generating functions as vectors in a Hilbert space H. And to do this in such a way that a finite selection of frequency bands will then correspond to a system of closed subspaces in H.

Spectral theory of the Fourier operator truncated on the positive half-axis

Victor Katsnelson, Weizmann Institute of Science

The tpectral theory of the Fourier operator truncated on the positive half-axis is developed.

Multivariable matrix-valued moment problems

David Kimsey, Weizmann Institute of Science

In this talk multivariable matrix-valued moment problems will be considered. Given a closed set $K \subseteq \mathbb{R}^d$ and a finite multisequence of Hermitian matrices $\{S_\gamma\}_{\gamma\in\Gamma}$, where, for example, $\Gamma = \{\gamma \in \mathbb{N}_0^d \colon 0 \le |\gamma| \le 2n+1\}$, we wish to determine whether or not there exists a positive semidefinite matrix-valued Borel measure Σ such that

$$\operatorname{supp} \Sigma \subseteq K \tag{1}$$

and

$$S_\gamma = \int_{\mathbb{R}^d} x^\gamma d\Sigma(x), \quad \gamma \in \Gamma. \tag{2}$$

We will provide concrete necessary and sufficient conditions for the case when $\{S_\gamma\}_{\gamma\in\Gamma}$ admits a minimal solution, that is when Σ is of the form $\sum_{j=1}^k P_j \delta_{x_j}$ with $\sum_{j=1}^k \operatorname{rank} P_j$ as small as possible. We will see that a similar result holds in the complex and bitorus setting. We will outline some applications of the above result.
A matrix-valued extension of Tchakaloff's theorem will be considered. Given a positive semidefinite matrix-valued Borel measure Σ, with card supp Σ which is infinite and having moments $\{S_\gamma\}_{\gamma\in\Gamma}$, where Γ is as above, can we find a positive semidefinite matrix-valued Borel measure with finite support, such that (1) and (2) hold? The answer turns out to be yes. We will see that such a result need not hold when Σ takes operators as values which act on an infinite dimensional Hilbert space.
This talk is partly based on joint work with H. J. Woerdeman.

Chaos expansion approach for optimal prediction of Gaussian processes

Alon Kipnis, Department of Mathematics, BGU

The optimal prediction problem of Gaussian processes refers to the problem of determining the statistics of some future sample of a Gaussian stochastic process given part of its past. This problem was first formulated and investigated by Wiener and Kolmogorov in the 40s of the previous century. In this talk we present an approach to that problem which is based on the Wiener chaos decomposition.
We begin with the Gaussian Hilbert space associated with a Gaussian stationary increment process. We then use properties of the Hardy space H^2 to construct a basis, which is based on the Laguerre functions, for the space of random variables measurable with respect to the same sigma field. This basis allows convenient representation of measurability with respect to the sub-sigma field generated by the past of the fundamental stationary increment process.

Centers and centroids of C*-algebras

Aldo Lazar, Tel-Aviv University

We shall discuss two topologies on the complete regularization, Glimm(A), of the primitive ideals space of a C*-algebra A and especially the compact subsets of it. Time permitting, we shall examine sufficient conditions for a C*-algebra to have a non trivial center.

Interpolation of Polynomials with Symmetries on the Imaginary Axis

Izchak Lewkowicz, ECE Department, BGU

One of the simplest version of interpolation problem is as follows: Given points $x_1\ ,\ \ldots\ ,\ x_m \in \mathbb{C}$ and $Y_1\ ,\ \ldots\ ,\ Y_m \in \mathbb{C}^{l\times l}$ find a minimal degree polynomial $F(s)$ ($F : \mathbb{C} \to \mathbb{C}^{l\times l}$) so that

$$F(x_j) = Y_j \qquad\qquad j = 1\ ,\ \ldots\ ,\ m.$$

We here mildly restrict the interpolation data and parametrize all minimal degree interpolating polynomials $F(s)$ admitting (pseudo) spectral factorization, i.e.

$$F(s) = G(s)G^*(-s^*)$$

for some polynomial $G(s)$.

The technique offered is easy to implement (and does not involve factorization). It appears that the recipe introduced here is applicable to a relatively large collection of interpolation problems. If time permits, we shall present versions where $F(s)$ admits J-(pseudo) spectral factorization or on the imaginary axis $F(s)$ has a Hamiltonian structure. In fact, the basic idea can be carried over to interpolating rational functions.

Joint work with D. Alpay, Math. dept. Ben-Gurion University.

The even gauge group of Powers' non-spatial E_0-semigroups

Daniel Markiewicz, Department of Mathematics, BGU

TBA

Mamadou Mboup, University of Reims Champagne–Ardenne

Operator Monotone Functions of Several Variables

John McCarthy, Washington University in St. Louis

Self-adjoint n-by-n matrices have a natural partial ordering, namely $A \leq B$ if the matrix $B - A$ is positive semi-definite.
In 1934 K. Loewner characterized functions that preserve this ordering; these functions are called n-matrix monotone. The condition depends on the dimension n, but if a function is n-matrix monotone for all n, then it must extend analytically to a function that maps the upper half-plane to itself.
I will describe Loewner's results, and then discuss what happens if one wants to characterize functions f of two (or more) variables that are matrix monotone in the following sense: If $A = (A_1, A_2)$ and $B = (B_1, B_2)$ are pairs of commuting self-adjoint n-by-n matrices, with $A_1 \leq B_1$ and $A_2 \leq B_2$, then $f(A) \leq f(B)$.
This talk is based on joint work with Jim Agler and Nicholas Young.

Endomorphisms vs. Transfer Operators: What's the difference?

Paul S. Muhly, University of Iowa, Iowa City, IA

I will speculate on relations between C^*-crossed products built from endomorphisms and C^*-algebras built using transfer operators. Concrete examples, connecting to fractals and wavelets, will be developed using groupoid technology.

C*-algebras and tracial $\mathcal{Z}$-absorption

Joav Orovitz, Department of Mathematics, BGU

Topological convolution algebras

Guy Salomon, Department of Mathematics, BGU

Let G be a locally compact group, with a Haar measure μ. Then, $L^1(G,\mu)$ is a convolution Banach-algebra (but not an Hilbert space). On the other hand, $L^2(G,\mu)$ is a Hilbert space, but is closed under convolution if and only if G is compact. In this talk we want to bridge the gap and present convolution algebras which behave locally as Hilbert spaces. More precisely, we introduce a new family of topological convolution algebras of the form $\bigcup_{p\in\mathbb{N}} L_2(G,\mu_p)$, which carries an inequality of the type $\|f*g\|_p \leq A_{p,q}\|f\|_q\|g\|_p$ for $p > q+d$ where d pre-assigned, and $A_{p,q}$ is a constant. We give a sufficient condition on the measures (μ_p) for such an inequality to hold, and show that the spectrum of any element in such an algebra is closed and is included in a specific disk in the complex plane. We also present two examples, one in the setting of non commutative stochastic distributions, and the other related to Dirichlet series. This is a joint work with Daniel Alpay.

Dilation theory in finite dimensions

Orr Shalit, Department of Mathematics, BGU

Traditionally, "dilation theory" belongs to the realm of operator theory on infinite dimensional spaces. Is there an effective version of dilation theory that does not require infinite dimensional spaces? If so, does this theory teach us something new? I hope to convince you that the answer to both questions is "yes". For example, I will present the following finite dimensional analogue of Ando's Theorem.

Theorem. *Let A and B be two commuting contractions on a finite dimensional Hilbert space H. Then for every N there exists two commuting unitaries V, U on a* **finite dimensional** *space $K \supseteq H$ such that for all $m, n \leq N$,*

$$A^m B^n = P_H V^m U^n P_H.$$

The talk is based on joint works with Eliahu Levy and John McCarthy.

Livsic overdetermined conservative systems on Lie Groups,

Eli Shamovich, Department of Mathematics, BGU

The study of overdetermined multidimensional systems was initiated by Livsic in [3] and then continued by many others, for example [1], [4] and [5]. The non-commutative setting was studied in a very general form by Gauchmann in [2] in term of connections on Hilbert bundles over a differentiable manifold. In this talk we will choose to concentrate on Lie groups, allowing us to consider invariant systems (analogues of time-invariant or constant coefficient systems in the classical 1D case) and to relate them to the representation theory of the group and of its Lie algebra. An example of a simple two-dimensional case will presented and worked out. This talk is based on joint work with V. Vinnikov.

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Abelian averages of discrete and continuous semigroups

David Shoikhet, Ort Braude College

(Joint work with Yuri Kozitsky and Jaroslav Zemánek)
Necessary and sufficient conditions are presented for the Abelian averages of discrete and continuous semigroups T^k and T_t to be power convergent in the operator norm topology in a complex Banach space. These conditions cover also the case where T is unbounded and the corresponding Abelian average is defined by means of the resolvent of T. These results have been generalized to the nonlinear holomorphic mappings on the Banach space and extend classical results by Michael Lin establishing sufficient conditions for the corresponding convergence for bounded T.

Composition operators on Sobolev spaces

Alexander Ukhlov, Ben-Gurion University

We study composition operators $\varphi^*(f) = f \circ \varphi$ on the Sobolev spaces of weakly differentiable functions in connection with the geometric function theory. As an application we obtain the Sobolev type embedding theorems and solvability of quasilinear elliptic equations.

Rational discrete analytic functions

Dan Volok, Kansas State University

The theory of discrete analytic functions has drawn a lot of attention recently, in part because of its connections with electrical networks and random walks. One aspect of this theory seems to cause significant difficulties: the pointwise product of two discrete analytic functions is not necessarily discrete analytic. In this talk we shall introduce a so-called C-K product of functions on the integer lattice, with the property that the C-K product of discrete analytic functions is discrete analytic, and investigate the class of the C-K quotients of discrete analytic polynomials.
This is a joint work with Daniel Alpay.