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A Problem of Enumeration of Two-color Bracelets with Several Variations

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Abstract. We consider the problem of enumeration of incongruent two-color bracelets of n beads, k of which are black, and study several natural variations of this problem. We also give recursion formulas for enumeration of t-color bracelets, $t \geq 3$.

1. Introduction

Professor Richard H. Reis (South-East University of Massachusetts, USA) in 1978 put the problem: "Let a circumference is split by the same n parts. It is required to find the number $R(n,k)$ of the incongruent convex k-gons, which could be obtained by connection of some k from n dividing points. Two k-gons are considered congruent if they are coincided at the rotation of one relatively other along the circumference and (or) by reflection of one of the k-gons relatively some diameter".

In 1979 Hansraj Gupta [1] gave the solution of the Reis problem.

Theorem 1 (H. Gupta).

$$R(n,k) = \frac{1}{2}\left(\binom{\lfloor \frac{n-h_k}{2} \rfloor}{\lfloor \frac{k}{2} \rfloor} + \frac{1}{k}\sum_{d|(k,n)} \varphi(d)\binom{\frac{n}{d}-1}{\frac{k}{d}-1}\right), \tag{1.1}$$

where $h_k \equiv k(\mod 2)$*,* $h_k = 0$ *or* 1*,* (n,k) *is* $\gcd(n,k)$*,* $\varphi(n)$ *- the Euler function.*

Consider some convex polygon with the tops in the circumference splitting points, "1" or "0" is put in accordance to each splitting point depending on whether a top of the polygon is in the point. Thus, there is the mutual one-to-one correspondence between the set of convex polygons with the tops in the circumference splitting points and the set of all $(0,1)$-configurations with the elements in these points.

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