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Research Article

Generalized Newman Phenomena and Digit Conjectures on Primes

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We prove that the ratio of the Newman sum over numbers multiple of a fixed integer, which is not a multiple of 3, and the Newman sum over numbers multiple of a fixed integer divisible by 3 is o(1) when the upper limit of summing tends to infinity. We also discuss a connection of our results with a digit conjecture on primes.

1. Introduction

Denote for $x, m \in \mathbb{N}$,

$$S_m(x) = \sum_{0 \le n < x,\, n \equiv 0 \pmod m} (-1)^{s(n)}, \tag{1.1}$$

where $s(n)$ is the number of 1's in the binary expansion of n. Sum (1.1) is a *Newman digit sum.* From the fundamental paper of Gelfond [1], it follows that

$$S_m(x) = O(x^\lambda), \quad \lambda = \frac{\ln 3}{\ln 4}. \tag{1.2}$$

The case $m = 3$ was studied in detail in [2–4].

So, from Coquet's theorem [3, 5] it follows that

$$-\frac{1}{3} + \frac{2}{\sqrt{3}} x^\lambda \le S_3(3x) \le \frac{1}{3} + \frac{55}{3}\left(\frac{3}{65}\right)^\lambda x^\lambda \tag{1.3}$$

with a microscopic improvement [4]:

$$\frac{2}{\sqrt{3}}x^{\lambda} \le S_3(3x) \le \frac{55}{3}\left(\frac{3}{65}\right)^{\lambda} x^{\lambda}, \quad x \ge 2, \tag{1.4}$$

and, moreover,

$$\left\lfloor 2\left(\frac{x}{6}\right)^{\lambda} \right\rfloor \le S_3(x) \le \left\lceil \frac{55}{3}\left(\frac{x}{65}\right)^{\lambda} \right\rceil. \tag{1.5}$$

These estimates give the most exact modern limits of the so-called *Newman phenomena*. Note that Drmota and Skałba [6], using a close function ($S_m^{(m)}(x)$), proved that if m is a multiple of 3, then for sufficiently large x,

$$S_m(x) > 0, \quad x \ge x_0(m). \tag{1.6}$$

In this paper, we study a general case for $m \ge 5$ (in the cases of $m = 2$ and $m = 4$, we have $|S_m(n)| \le 1$).

To formulate our results, put for $m \ge 5$,

$$\lambda_m = 1 + \log_2 b_m, \tag{1.7}$$

$$\mu_m = \frac{2}{2b_m - 1}, \tag{1.8}$$

where

$$b_m^2 = \begin{cases} \sin\left(\frac{\pi}{3}\left(1 + \frac{3}{m}\right)\right)\left(\sqrt{3} - \sin\left(\frac{\pi}{3}\left(1 + \frac{3}{m}\right)\right)\right), & \text{if } m \equiv 0 \pmod 3, \\ \sin\left(\frac{\pi}{3}\left(1 - \frac{1}{m}\right)\right)\left(\sqrt{3} - \sin\left(\frac{\pi}{3}\left(1 - \frac{1}{m}\right)\right)\right), & \text{if } m \equiv 1 \pmod 3, \\ \sin\left(\frac{\pi}{3}\left(1 + \frac{1}{m}\right)\right)\left(\sqrt{3} - \sin\left(\frac{\pi}{3}\left(1 + \frac{1}{m}\right)\right)\right), & \text{if } m \equiv 2 \pmod 3. \end{cases} \tag{1.9}$$

Directly, one can see that

$$\frac{\sqrt{3}}{2} > b_m \ge \begin{cases} 0.86184088\cdots, & \text{if } (m,3) = 1, \\ 0.85559967\cdots, & \text{if } (m,3) = 3, \end{cases} \tag{1.10}$$

and thus,

$$\lambda_m < \lambda,$$

$$2.73205080\cdots < \mu_m \leq \begin{cases} 2.76364572\cdots, & \text{if } (m,3) = 1, \\ 2.81215109\cdots, & \text{if } (m,3) = 3. \end{cases} \tag{1.11}$$

Below, we prove the following results.

Theorem 1.1. *If* $(m,3) = 1$, *then*

$$\left|S_m(x)\right| \leq 1 + \mu_m x^{\lambda_m}. \tag{1.12}$$

Theorem 1.2 (Generalized Newman phenomena). *If* $m > 3$ *is a multiple of 3, then*

$$\left|S_m(x) - \frac{3}{m}S_3(x)\right| \leq 1 + \mu_m x^{\lambda_m}. \tag{1.13}$$

Using Theorem 1.2 and (1.5), one can estimate $x_0(m)$ in (1.6). For example, one can prove that $x_0(21) < e^{909}$.

2. Explicit formula for $S_m(N)$

We have

$$\begin{aligned} S_m(N) &= \sum_{n=0,\, m|n}^{N-1} (-1)^{s(n)} \\ &= \frac{1}{m}\sum_{t=0}^{m-1}\sum_{n=0}^{N-1}(-1)^{s(n)} e^{2\pi i(nt/m)} \\ &= \frac{1}{m}\sum_{t=0}^{m-1}\sum_{n=0}^{N-1} e^{2\pi i((t/m)n+(1/2)s(n))}. \end{aligned} \tag{2.1}$$

Note that the interior sum has the form

$$F_\alpha(N) = \sum_{n=0}^{N-1} e^{2\pi i(\alpha n+(1/2)s(n))}, \quad 0 \leq \alpha < 1. \tag{2.2}$$

Lemma 2.1. *If* $N = 2^{\nu_0} + 2^{\nu_1} + \cdots + 2^{\nu_r}$, $\nu_0 > \nu_1 > \cdots > \nu_r \geq 0$, *then*

$$F_\alpha(N) = \sum_{h=0}^{r} e^{2\pi i(\alpha\sum_{j=0}^{h-1}2^{\nu_j}+h/2)} \prod_{k=0}^{\nu_h-1}\left(1 + e^{2\pi i(\alpha 2^k+1/2)}\right), \tag{2.3}$$

where as usual $\sum_{j=0}^{-1} = 0$, $\prod_{k=0}^{-1} = 1$.

Proof. Let $r = 0$, then by (2.2),

$$\begin{aligned} F_\alpha(N) &= \sum_{n=0}^{N-1} (-1)^{s(n)} e^{2\pi i \alpha n} \\ &= 1 - \sum_{j=0}^{\nu_0 - 1} e^{2\pi i \alpha 2^j} + \sum_{0 \le j_1 < j_2 \le \nu_0 - 1} e^{2\pi i \alpha (2^{j_1} + 2^{j_2})} - \cdots \\ &= \prod_{k=0}^{\nu_0 - 1} \left(1 - e^{2\pi i \alpha 2^k}\right), \end{aligned} \tag{2.4}$$

which corresponds to (2.3) for $r = 0$.

Assuming that (2.3) is valid for every N with $s(N) = r + 1$, let us consider $N_1 = 2^{\nu_r} a + 2^{\nu_{r+1}}$ where a is odd, $s(a) = r + 1$, and $\nu_{r+1} < \nu_r$. Let

$$\begin{aligned} N &= 2^{\nu_r} a = 2^{\nu_0} + \cdots + 2^{\nu_r}; \\ N_1 &= 2^{\nu_0} + \cdots + 2^{\nu_r} + 2^{\nu_{r+1}}. \end{aligned} \tag{2.5}$$

Notice that for $n \in [0, 2^{\nu_{r+1}})$, we have

$$s(N + n) = s(N) + s(n). \tag{2.6}$$

Therefore,

$$\begin{aligned} F_\alpha(N_1) &= F_\alpha(N) + \sum_{n=N}^{N_1 - 1} e^{2\pi i (\alpha n + (1/2) s(n))} \\ &= F_\alpha(N) + \sum_{n=0}^{2^{\nu_{r+1}} - 1} e^{2\pi i (\alpha n + \alpha N + (1/2)(s(N) + s(n)))} \\ &= F_\alpha(N) + e^{2\pi i (\alpha N + (1/2) s(N))} \sum_{n=0}^{2^{\nu_{r+1}} - 1} e^{2\pi i (\alpha n + (1/2) s(n))}. \end{aligned} \tag{2.7}$$

Thus, by (2.3) and (2.4),

$$\begin{aligned} F_\alpha(N_1) &= \sum_{h=0}^{r} e^{2\pi i (\alpha \sum_{j=0}^{h-1} 2^{\nu_j} + h/2)} \prod_{k=0}^{\nu_h - 1} \Big(1 + e^{2\pi i (\alpha 2^k + 1/2)} \\ &\quad + e^{2\pi i (\alpha \sum_{j=0}^{r} 2^{\nu_j} + (r+1)/2)} \prod_{k=0}^{\nu_{r+1} - 1} \left(1 + e^{2\pi i (\alpha 2^k + 1/2)}\right) \\ &= \sum_{h=0}^{r+1} e^{2\pi i (\alpha \sum_{j=0}^{h-1} 2^{\nu_j} + h/2)} \prod_{k-0}^{\nu_h - 1} \Big(1 + e^{2\pi i (\alpha 2^k + 1/2)}\Big). \end{aligned} \tag{2.8}$$

□

Formulas (2.1)–(2.3) give an explicit expression for $S_m(N)$ as a linear combination of the products of the form

$$\prod_{k=0}^{\nu_h-1}\left(1+e^{2\pi i(\alpha 2^k+1/2)}\right), \quad \alpha=\frac{t}{m},\ 0\le t\le m-1. \tag{2.9}$$

Remark 2.2. On can extract (2.3) from a very complicated general Gelfond formula [1], however, we prefer to give an independent proof.

3. Proof of Theorem 1.1

Note that in (2.3)

$$r\le \nu_0=\left\lfloor\frac{\ln N}{\ln 2}\right\rfloor. \tag{3.1}$$

By Lemma 2.1, we have

$$\begin{aligned}\left|F_\alpha(N)\right| &\le \sum_{\nu_h=\nu_0,\nu_1,\ldots,\nu_r}\left|\prod_{k=1}^{\nu_h}\left(1+e^{2\pi i(\alpha 2^{k-1}+1/2)}\right)\right| \\ &\le \sum_{h=0}^{\nu_0}\left|\prod_{k=1}^{h}\left(1+e^{2\pi i(\alpha 2^{k-1}+1/2)}\right)\right|.\end{aligned} \tag{3.2}$$

Furthermore,

$$1+e^{2\pi i(2^{k-1}\alpha+1/2)}=2\sin\left(2^{k-1}\alpha\pi\right)\left(\sin\left(2^{k-1}\alpha\pi\right)-i\cos\left(2^{k-1}\alpha\pi\right)\right) \tag{3.3}$$

and, therefore,

$$\left|1+e^{2\pi i(2^{k-1}\alpha+1/2)}\right|\le 2\left|\sin\left(2^{k-1}\alpha\pi\right)\right|. \tag{3.4}$$

According to (3.2), let us estimate the product

$$\prod_{k=1}^{h}\left(2\left|\sin\left(2^{k-1}\alpha\pi\right)\right|\right)=2^h\prod_{k=1}^{h}\left|\sin\left(2^{k-1}\alpha\pi\right)\right|, \tag{3.5}$$

where by (2.1),

$$\alpha=\frac{t}{m}, \quad 0\le t\le m-1. \tag{3.6}$$

Repeating arguments of [1], put

$$\left|\sin\left(2^{k-1}\alpha\pi\right)\right| = t_k. \tag{3.7}$$

Considering the function

$$\rho(x) = 2x\sqrt{1-x^2}, \quad 0 \le x \le 1, \tag{3.8}$$

we have

$$t_k = 2t_{k-1}\sqrt{1-t_{k-1}^2} = \rho\left(t_{k-1}\right). \tag{3.9}$$

Note that

$$\rho'(x) = 2\left(\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}\right) \le -1 \tag{3.10}$$

for $x_0 \le x \le 1$, where

$$x_0 = \frac{\sqrt{3}}{2} \tag{3.11}$$

is the only positive root of the equation $\rho(x) = x$.

Show that either

$$t_k \le \sin\left(\frac{\pi}{m}\left\lfloor\frac{m}{3}\right\rfloor\right) = \sin\left(\frac{\pi}{m}\left\lceil\frac{2m}{3}\right\rceil\right) = g_m < \frac{\sqrt{3}}{2} \tag{3.12}$$

or, simultaneously, $t_k > g_m$, and

$$\begin{aligned} t_k t_{k+1} &\le \max_{0\le l\le m-1}\left(\left|\sin\frac{l\pi}{m}\right|\left(\sqrt{3}-\left|\sin\frac{l\pi}{m}\right|\right)\right) \\ &= \begin{cases} \left(\sin\left(\dfrac{\pi}{m}\left\lfloor\dfrac{m}{3}\right\rfloor\right)\right)\left(\sqrt{3}-\sin\left(\dfrac{\pi}{m}\left\lfloor\dfrac{m}{3}\right\rfloor\right)\right), & \text{if } m \equiv 1 \pmod 3 \\ \left(\sin\left(\dfrac{\pi}{m}\left\lceil\dfrac{m}{3}\right\rceil\right)\right)\left(\sqrt{3}-\sin\left(\dfrac{\pi}{m}\left\lceil\dfrac{m}{3}\right\rceil\right)\right), & \text{if } m \equiv 2 \pmod 3 \end{cases} = h_m < \frac{3}{4}. \end{aligned} \tag{3.13}$$

Indeed, let for a fixed values of $t \in [0, m-1]$ and $k \in [1, n]$,

$$t2^{k-1} \equiv l (\operatorname{mod} m), \quad 0 \le l \le m-1. \tag{3.14}$$

Then,

$$t_k = \left| \sin \frac{l\pi}{m} \right|. \tag{3.15}$$

Now, distinguish two cases: (1) $t_k \le \sqrt{3}/2$, (2) $t_k > \sqrt{3}/2$.

In case (1),

$$t_k = \frac{\sqrt{3}}{2} \leftrightarrows \frac{l\pi}{m} = \frac{r\pi}{3}, \quad (r, 3) = 1, \tag{3.16}$$

and since $0 \le l \le m - 1$, then

$$m = \frac{3l}{r}, \quad r = 1, 2. \tag{3.17}$$

Because of the condition $(m, 3) = 1$, we have $t_k < \sqrt{3}/2$.

Thus, in (3.15),

$$l \in \left[0, \left[\frac{m}{3}\right]\right] \cup \left[\left[\frac{2m}{3}\right], m\right], \tag{3.18}$$

and (3.12) follows.

In case (2), let $t_k > \sqrt{3}/2 = x_0$. For $\varepsilon > 0$, put

$$1 + \varepsilon = \frac{t_k}{x_0} = \frac{2}{\sqrt{3}} \left| \sin\left(\pi 2^{k-1} \alpha\right) \right| \tag{3.19}$$

such that

$$1 - \varepsilon = 2 - \frac{2}{\sqrt{3}} \left| \sin\left(\pi 2^{k-1} \alpha\right) \right|, \tag{3.20}$$

$$1 - \varepsilon^2 = \frac{4}{3} \left| \sin\left(\pi 2^{k-1} \alpha\right) \right| \left(\sqrt{3} - \left| \sin\left(\pi 2^{k-1} \alpha\right) \right|\right). \tag{3.21}$$

By (3.9) and (3.19), we have

$$t_{k+1} = \rho(t_k) = \rho((1 + \varepsilon)x_0) = \rho(x_0) + \varepsilon x_0 \rho'(c), \tag{3.22}$$

where $c \in (x_0, (1 + \varepsilon)x_0)$.

Thus, according to (3.10) and taking into account that $\rho(x_0) = x_0$, we find

$$t_{k+1} \le x_0(1 + \varepsilon), \tag{3.23}$$

while by (3.19)

$$t_k = x_0(1+\varepsilon). \tag{3.24}$$

Now, in view of (3.21) and (3.11),

$$t_k t_{k+1} \leq \left|\sin \pi 2^{k-1}\alpha\right|\left(\sqrt{3} - \left|\sin\left(\pi 2^{k-1}\alpha\right)\right|\right), \tag{3.25}$$

and according to (3.14), (3.15), we obtain that

$$t_k t_{k+1} \leq h_m, \tag{3.26}$$

where h_m is defined by (3.13).

Notice that from simple arguments and according to (1.9),

$$g_m \leq \sqrt{h_m} = b_m. \tag{3.27}$$

Therefore,

$$\prod_{k=1}^{h}\left|\sin\left(\pi 2^{k-1}\alpha\right)\right| \leq \left(b_m^{\lfloor h/2\rfloor}\right)^2 \leq b_m^{h-1}. \tag{3.28}$$

Now, by (3.2)-(3.4), for $\alpha = t/m,\ t = 0, 1, \dots, m-1$, we have

$$\begin{aligned}
\left|F_{t/m}(N)\right| &\leq \sum_{h=0}^{\nu_0}\left|\prod_{k=1}^{h}\left(1 + e^{2\pi i(\alpha 2^{k-1}+1/2)}\right)\right| \\
&\leq \sum_{h=0}^{\nu_0} 2^h \prod_{k=1}^{h}\left|\sin\left(2^{k-1}\alpha\pi\right)\right| \\
&\leq 1 + 2\sum_{h=1}^{\nu_0}\left(2b_m\right)^{h-1} \\
&\leq 1 + 2\frac{\left(2b_m\right)^{\nu_0}}{2b_m - 1}.
\end{aligned} \tag{3.29}$$

Note that, according to (1.7) and (3.1),

$$\left(2b_m\right)^{\nu_0} = 2^{\lambda_m \nu_0} \leq 2^{\lambda_m \log_2 N} = N^{\lambda_m}. \tag{3.30}$$

Thus, by (1.8)

$$\left|F_{t/m}(N)\right| \leq 1 + \frac{2}{2b_m - 1}N^{\lambda_m} = 1 + \mu_m N^{\lambda_m}. \tag{3.31}$$

Thus, the theorem follows from (2.1).

4. Proof of Theorem 1.2

Select in (2.1) the summands which correspond to $t = 0,\ m/3,\ 2m/3$.

We have

$$mS_m(N) = \sum_{n=0}^{N-1}\left(e^{\pi i s(n)} + e^{2\pi i(n/3+(1/2)s(n))} + e^{2\pi i(2n/3+(1/2)s(n))}\right) + \sum_{t=1,\,t\,\neq\, m/3,2m/3}^{m-1}\ \sum_{n=0}^{N-1} e^{2\pi i((t/m)n+(1/2)s(n))}. \tag{4.1}$$

Since the chosen summands do not depend on m and, for $m = 3$, the latter sum is empty, then we find

$$mS_m(N) = 3S_3(N) + \sum_{t=1,\,t\,\neq\, m/3,2m/3}^{m-1}\ \sum_{n=0}^{N-1} e^{2\pi i((t/m)n+(1/2)s(n))}. \tag{4.2}$$

Further, the last double sum is estimated by the same way as in Section 3 such that

$$\left|S_m(N) - \frac{3}{m}S_3(N)\right| \leq \mu_m N^{\lambda_m}. \tag{4.3}$$

Remark 4.1. Notice that from elementary arguments it follows that if $m \geq 5$ is a multiple of 3, then

$$\left(\sin\frac{\pi}{m}\left\lfloor\frac{m-1}{3}\right\rfloor\right)\left(\sqrt{3} - \sin\frac{\pi}{m}\left\lfloor\frac{m-1}{3}\right\rfloor\right) \leq \left(\sin\frac{\pi}{m}\left\lceil\frac{m+1}{3}\right\rceil\right)\left(\sqrt{3} - \sin\frac{\pi}{m}\left\lceil\frac{m+1}{3}\right\rceil\right). \tag{4.4}$$

The latter expression is the value of b_m^2 in this case (see (1.9)).

Example 4.2. Let us find some x_0 such that $S_{21}(x) > 0$ for $x \geq x_0$.

Supposing that x is multiple of 3 and using (1.4), we obtain that

$$S_3(x) \geq \frac{2}{3^{\lambda+1/2}}x^{\lambda}. \tag{4.5}$$

Therefore, putting $m = 21$ in Theorem 1.2, we have

$$S_{21}(x) \geq \frac{1}{7}S_3(x) - \mu_{21}x^{\lambda_{21}} - 1 \geq \frac{2}{7\cdot 3^{\lambda+1/2}}x^{\lambda} - \mu_{21}x^{\lambda_{21}} - 1. \tag{4.6}$$

Now, calculating λ and λ_m by (1.2) and (1.8), we find a required x_0:

$$x_0 = \left(3.5\cdot 3^{\lambda+1/2}\mu_{21}\right)^{1/(\lambda-\lambda_{21})} = e^{908.379\ldots}. \tag{4.7}$$

Corollary 4.3. *For m which is not a multiple of 3, denote $U_m(x)$ the set of the positive integers not exceeding x which are multiples of m and not multiples of 3. Then,*

$$\sum_{n\in U_m(x)} (-1)^{s(n)} = -\frac{1}{m}S_3(x) + O\left(x^{\lambda_m}\right). \tag{4.8}$$

In particular, for sufficiently large x, we have

$$\sum_{n\in U_m(x)} (-1)^{s(n)} < 0. \tag{4.9}$$

Proof. Since

$$\left|U_m(x)\right| = S_m(x) - S_{3m}(x), \tag{4.10}$$

then the corollary immediately follows from Theorems 1.1, 1.2. □

5. On Newman sum over primes

In [7], we put the following binary digit conjectures on primes.

Conjecture 5.1. For all $n \in \mathbb{N}$, $n \neq 5, 6$,

$$\sum_{p\le n}(-1)^{s(p)} \le 0, \tag{5.1}$$

where the summing is over all primes not exceeding n.

More precisely, by the observations, $\sum_{p\le n}(-1)^{s(p)} < 0$ beginning with $n = 31$. Moreover, the following conjecture holds.

Conjecture 5.2.

$$\lim_{n\to\infty} \frac{\ln\left(-\sum_{p\le n}(-1)^{s(p)}\right)}{\ln n} = \frac{\ln 3}{\ln 4}. \tag{5.2}$$

A heuristic proof of Conjecture 5.2 was given in [8]. For a prime p, denote $V_p(x)$ the set of positive integers not exceeding x for which p is the least prime divisor. Show that the correctness of Conjectures 5.1 (for $n \ge n_0$) follows from the following very plausible statement, especially in view of the above estimates.

Conjecture 5.3. For sufficiently large n, we have

$$\begin{aligned}\left|\sum_{5\le p\le\sqrt{n}}\ \sum_{j\in V_p(n),\, j>p} (-1)^{s(j)}\right| &< \sum_{j\in V_3(n)} (-1)^{s(j)} \\ &= S_3(n) - S_6(n).\end{aligned} \tag{5.3}$$

Indeed, in the "worst case" (really is not satisfied), in which for all $n \geq p^2$

$$\sum_{j \in V_p(n),\, j>p} (-1)^{s(j)} < 0, \quad p \geq 5, \tag{5.4}$$

we have a decreasing but *positive* sequence of sums:

$$\begin{gathered} \sum_{j \in V_3(n),\, j>3} (-1)^{s(j)}, \sum_{j \in V_3(n),\, j>3} (-1)^{s(j)} + \sum_{j \in V_5(n),\, j>5} (-1)^{s(j)}, \ldots, \\ \sum_{j \in V_3(n),\, j>3} (-1)^{s(j)} + \sum_{5 \leq p < \sqrt{n}} \sum_{j \in V_p(n),\, j>p} (-1)^{s(j)} > 0. \end{gathered} \tag{5.5}$$

Hence, the "balance condition" for odd numbers [8]

$$\left| \sum_{j \leq n,\ j \text{ is odd}} (-1)^{s(j)} \right| \leq 1 \tag{5.6}$$

must be ensured permanently by the excess of the odious primes. This explains Conjecture 5.1.

It is very interesting that for some primes p the inequality (5.4), indeed, is satisfied for all $n \geq p^2$. Such primes we call "resonance primes." Our numerous observations show that all resonance primes not exceeding 1000 are

$$\begin{gathered} 11, 19, 41, 67, 107, 173, 179, 181, 307, 313, 421, 431, 433, 587, \\ 601, 631, 641, 647, 727, 787. \end{gathered} \tag{5.7}$$

In conclusion, note that for $p \geq 3$, we have

$$\lim_{n \to \infty} \frac{|V_p(n)|}{n} = \frac{1}{p} \prod_{2 \leq q < p} \left(1 - \frac{1}{q}\right) \tag{5.8}$$

such that

$$\lim_{n \to \infty} \left(\sum_{p \geq 3} \frac{|V_p(n)|}{n} \right) = \frac{1}{2}. \tag{5.9}$$

Thus, using Theorems 1.1, 1.2 in the form

$$S_m(n) = \begin{cases} o(S_3(n)), & (m,3) = 1, \\ \dfrac{3}{m} S_3(n)(1 + o(1)), & 3 \mid m, \end{cases} \tag{5.10}$$

and inclusion-exclusion for $p \geq 5$, we find

$$\begin{aligned}\sum_{j\in V_p(n)} (-1)^{\sigma(j)} &= -\frac{3}{3p}\prod_{2\leq q<p,\, q\,\neq\, 3}\left(1-\frac{1}{q}\right)S_3(n)(1+o(1)) \\ &= -\frac{3}{2p}\prod_{2\leq q<p}\left(1-\frac{1}{q}\right)S_3(n)\bigl(1+o(1)\bigr).\end{aligned} \tag{5.11}$$

Now, in view of (1.5), we obtain the following absolute result as an approximation of Conjectures 5.1, 5.2.

Theorem 5.4. *For every prime number $p \geq 5$ and sufficiently large $n \geq n_p$, we have*

$$\sum_{j\in V_p(n)} (-1)^{s(j)} < 0 \tag{5.12}$$

and, moreover,

$$\lim_{n\to\infty}\frac{\ln\Bigl(-\sum_{j\in V_p(n)}(-1)^{s(j)}\Bigr)}{\ln n} = \frac{\ln 3}{\ln 4}. \tag{5.13}$$

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Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from "Qualitative Theory of Differential Equations," allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the *Mathematical Problems in Engineering* aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

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