MAIN ACHIEVEMENTS IN DISCRETE MATHEMATICS

AND CODING THEORY

Solutions and generalizations of some known problems

a) Theory of permutations in Combinatorics

1. Wide generalization of well-known Riordan(1944)-Moser(1982) formulas for Latin rectangles: [13],[17],[18],[22, pp.255-257].
2. A conjecture of K. Yamamoto (1956) from his paper "Structure polynomial of Latin rectangles and its application to a combinatorial problem. *Mem. Fac. Kyusyu Univ*., Ser. A (1956), 10, 1-13. Solution: [8].
3. Minc`s problem on representations of permanents of circulants (1962) in form of power sums (cf. in his best known book "Permanents", Ch. 3, formulae 4.7, 4.9, 4.10).
   Solution and wide generalization : [11], [21, Section 4].
4. Enumeration of permutations with restrict position and a fixed number of cycles: [19], [21].
5. Problem of Greenstein (1978) from famous "Minc`s List", № 3. [22], pp.258-259 (see details of solution algorithm in arXiv:1104.4051)
6. Conjecture of K. Balasubramanian (1990) connected with the famous H.Ryser`s conjecture (1967). Disproof: [22], pp.249-250.
7. Counter-example to Nicol's formula for the number of solutions of the congruence $x_1 + x_2 + ... + x_s \equiv r(\mathrm{mod}\ k)$. It is obtained combinatorial (with the Gaus Polynomials) and algebraic solutions of this problem.
   New formula in case of distinct x's is
   sum{d|(k,s)}(-1)^(s+s/d)C(k/d,s/d)Φ(r,d)/k, where C(m,n) is binomial coefficient, Φ (m,n)=phi(n)mu(n/(m,n))/phi(n/(m,n)). See [30].
8. Short Gupta's enumeration of convex k-gons (see H. Gupta, Indian J. pure appl. Math. 10(8)(1979), 964-999). [35].
9. Analog of classic Erdos-Renyi theorem for stochastic (0,1)-matrices with equal row sums (cf. Studia Sci. Math. Hung. 3(4) (1968), 459-464). [31].
10. Discovery of simple algorithms and recursive formulae for enumerating the permutations with prescribed up-down structure using a function of two variables. [46].

b) Theory of numbers

A new field in number theory:

11. "Fermi – Dirac Number arithmetic". The pioneer author's paper (1981) is [6] (MR0632989(83a:10003)) ; a further development contains in [26], see also a large review in MR 2000f: 11097, pp. 3912-3913. The duality of usual arithmetic and Fermi-Dirac one essentially is similar to the known duality of Bose-Einstein and Fermi-Dirac statistics for particles. Later and independently G. L. Cohen (1990) introduced "infinitary arithmetic" which is equivalent to the Fermi-Dirac one. A generalization and further development see in [40] .
12. Extension of the fundamental Fermat theorem on powers of irrational numbers: [37] (2005).
13. Full solution in [39] (2007) of a new Diophantine equation $p^x+p^y+p^z=2^u+3$ in nonnegative integers x,y,z,u, where p is an arbitrary Fermat prime (see in [39] pp. 215-224). This allows to investigate introduced by the author so-called "compact numbers" in the frameworks of the natural continuation of Berend's known research of factorials answered on Erdos-Graham question.
14. Theorem ([41] (2008)). For every prime $p\geq 5$ and $n\geq n(p)$, among the first n positive integers with the least divisor p, the numbers with odd 1's in its binary representation are always in the majority. Conjecture. For all $n\geq 6$, among primes $p\leq n$, the primes with odd 1's in its binary representation are always in the majority.
15. Exact solution in [42] (2009) of the problem of the exponent in remainder term of Gelfond's digit theorem (in binary case). For the lower limit, see paper by I. Shparlinski "On the size of the Gelfond exponent", *J. Number Theory*, 2010, v.130, 1056-1060.
16. Proof of Stephan's conjectures concerning Pascal triangle modulo 2 and their polynomial generalization: [48].
17. Introducing and studying in [50] (2012) so-called "overpseudoprimes" to b which are stronger than strong pseudoprimes to the same base. It was proven that many best known numbers are primes or overpseudoprimes (for example, Fermat numbers $2^{(2^n)}+1$, Mersenne numbers 2^p-1 with prime p, squares of Wieferich primes are primes or

overpseudoprimes to base 2).

c) Problems of S. Ramanujan (1912)

18. Solution [32] and quite new formulae [43] (for example, see astonishing formulae (31)-(32)). For further development, see papers by R. Witula in *J. of Integer Seq*. Vol. 13 (2010) , articles 10.5.7 and 10.7.5.

d) Coding theory

19. Solution ([33] (2002)) of the problem of estimation of the average distance distribution (or weight enumerator function) of the different types (we give eight examples) of regular ensembles of low-density parity-check codes. The problem was addressed in many papers starting with R. G. Gallager (1963), but the average distance distribution was unknown even for the ensemble of codes defined by the parity-check matrices having fixed (and equal) number of 1's in every row and column.
20. Solution ([34](2003) of the previous problem for some irregular ensembles of low-density parity-check codes.