NETWORKS OF LINEAR SYSTEMS AND OPERATOR THEORY
SDE BOKER, MARCH 19-24, 2017

ABSTRACTS OF TALKS

Arov Damir. *Minimal passive state/signal realizations of passive behaviors with minimal scattering channel losses.*

We consider regular passive linear continuous time state/signal systems $\Sigma = (V; X, W)$ with separable Hilbert state space X, Krein signal space W, and the maximal nonnegative generating subspace V in the Krein node space $K = X \times X \times W$ with the inner product defined by the indefinite quadratic form

$$\left\langle \begin{bmatrix} z \\ x \\ w \end{bmatrix}, \begin{bmatrix} z \\ x \\ w \end{bmatrix} \right\rangle = -2\Re(z, x)_X + [w, w]_W, \quad \begin{bmatrix} z \\ x \\ w \end{bmatrix} \in K.$$

Certain additional regularity condition are imposed on the generating subspace V that guarantee that for a dense subspace X^o in X there exist a sufficiently rich set of future classical trajectories

$$(x^o, x(t), w(t)) \text{ of } \Sigma \text{ i.e. } x \in \mathcal{C}^1(R_+; W), w \in \mathcal{C}(R_+; W), x(0) = x^o \in X^o,$$

and

$$\begin{bmatrix} \dot{x}(t) \\ x(t) \\ w(t) \end{bmatrix} \in V, t \in R_+.$$

Through approximation by such classical trajectories one may then construct a sufficiently rich set of generalized trajectories $(x^o, x(t), w(t))$ of Σ satisfying $x \in \mathcal{C}(R_+; X)$ and $\omega \in L_2(R_+; W)$ with arbitrary initial states $x^o \in X$. The subspace $\mathcal{R}_+$ of $L_2(R_+; W)$ of the signal components of generalized trajectories of Σ with zero initial state is a maximal nonnegative shift invariant (closed) subspace of the Krein space $K_2(R_+; W)$, which as topological vector space coincides with $L_2(R_+;)$ but which is equipped with the indefinite inner product inherited from the Krein space W. This subspace is called the future behavior of Σ and any subspace $\mathcal{R}_+$ with these properties is called a *passive future behavior* on the Krein signal space W.

In recent work we have shown that any passive future behavior on a Krein space may be realized as the future behavior of a conservative state/signal system Σ, i.e., a passive state/signal system with a Lagrangian (hypermaximal neutral) generating subspace V (i.e. $V = V^{([\perp])}$) with the additional property that Σ does not have a proper conservative compression. Such a system is called a *simple conservative state/signal system*, and it is defined by its future behavior up to unitary similarity. The *internal scattering channels* of such a system has in a certain sense *minimal losses.* By compressing such a system one gets minimal passive realizations of the same behavior (minimality meaning that these systems cannot be compressed any further) with minimal losses of scattering channels.

The notions of internal and external scattering channels of a passive system will be discussed in this talk and it will be explained what it means for these channels to have minimal losses. If W is not a Hilbert or anti-Hilbert space, then it has infinite many fundamental decompositions $W = U[+] - Y$. To each such decomposition corresponds a passive (input/state/output) scattering system $\Sigma_{sc} = (S; X, U, Y)$, and this system is conservative or minimal passive with minimal losses if and only if the original state/signal system has the same properties. Also to every other direct sum decompositions $W = U\dot{+}Y$ of W (Lagrangian, or orthogonal, or a general decomposition) corresponds a passive i/s/o system representation whose supply rates is not of scattering type (but instead of impedance, or transmission, or other type), and also the systems obtained in this way are simple conservative or minimal passive i/s/o systems with same scattering channels with minimal losses (with respect to the appropriate supply rate). This talk is based on joint work with Dr. Mikael Kurula and Prof. Olof Staffans, and it is intimately connected to the results presented in Olof Staffans talk on state/signal invariant properties of input/state/output systems.

Ball Joe. *Standard versus strict bounded real lemma with infinite-dimensional state space.*

The Bounded Real Lemma, i.e., the state-space LMI characterization (referred to as the Kalman-Yakubovich-Popov or KYP inequality) of when an input/state/output linear system satisfies a dissipation inequality, has recently been studied for infinite-dimensional discrete-time systems in a number of different settings (with or without stability assumptions, with or without controllability/observability assumptions, with or without strict inequalities). In these various settings, sometimes unbounded solutions of the KYPO inequality are required while in other instances bounded solutions suffice. We saw how these diverse results can be reconciled by making a careful distinction between *standard Bounded Real Lemma* where one allows nonstrict versions of the KYP inequality, and *strict Bounded Real Lemma* where one insists that insists that any solution satisfy the KYP condition with strict inequality. As an application one can derive a a strict KYP-inequality characterization for strictly contractive time-varying input-output maps on ℓ_2.

Brockett Roger. *Synchronization is not just for Oscillations.*

Synchronization of oscillations has been studied since the time of Huygens in the 1600s. More recently it has come to prominence in connection with communications technology and multi-body physics. In an earlier paper dedicated to Uwe Helmke I described a simple mechanism for achieving synchronization of systems of classic, second order harmonic oscillators, as opposed to the phase only model of Kuramoto. Although not as widely studied as their physical incarnations, oscillations play an important role in cell biology, acting along with elaborate sequencing mechanisms enabling cell division. To capture this mathematically what seems to be needed is a theory that acknowledges both synchronization and precision sequencing of events. This talk will describe recent progress in this area building on previous work on synchronization.

Jian Hua Chen *Time-varying multiplicative perturbations of well-posed linear systems*

Abstract: We study a time-varying well-posed system resulting from multiplicative perturbation of the generator of a time-invariant well-posed system. The associated generator family takes the form AP(t), where P(t) is a bounded operator on the state space and the operator-valued function P is strongly continuous. This is motivated by the physical system involving electromagnetic field around a moving object governed by Maxwell's equations. The problem has not been solved completely and the challenging difficulty is addressed.

Cohen Guy. *Rates of convergence of powers of contractions.*

We discuss different conditions on the numerical range $W(T) = \{\langle Tf, f\rangle : \|f\| = 1\}$ of a Hilbert space contraction T which yield a rate of convergence $\|T^n(I-T)\| = \mathcal{O}(1/n^\beta)$, $\beta \in [\frac{1}{2}, 1)$. As a consequence of recent work by Seifert, a necessary and sufficient condition for the above rate for T power-bounded is given by the growth rate $\sup_{|\lambda|>1} |\lambda - 1|^{1/\beta}\|R(\lambda, T)\| < \infty$. When T is a contraction on L^2 satisfying the numerical range condition, we obtain that $T^n f/n^{1-\beta}$ converges a.e. to 0 with a maximal inequality, for every $f \in L^2$.
This is a joint work with Michael Lin.

Dewilde Patrick. *Polynomial system models with a non-commuting shift.*

Can the classical matrix-fraction models for linear, time invariant (LTI) systems build with polynomials-in-the-shift be extended to systems where the 'natural shift does not commute with system operators, as is the case with time-variant or non-linear systems? The usual Popov- Euclidean constructions do not apply anymore, but a system-theoretic approach remains possible. It not only leads to adequate models, but also produces Bezout identities so that most of the classical LTI theory extends, although the module structure is lost in favor of just a nest algebra. The theory finds attractive applications, in particular the solution of the Loewner interpolation problem for this general context, which allows for model reduction directly on a given data set (as has been advocated by A.C. Antoulas).

Dirr Gunther. *Parallel Connections of Bilinear Systems: the Simplest Case of a Network.*

First, we will develop necessary and sufficient conditions for accessibility and controllability of fnitely many parallel connected bilinear systems. The key assumption will be that the underlying (matrix-) Lie group on which the single bilinear systems evolve is simple. Under this hypothesis we can derive a "simple" accessibility condition for their parallel connection which is based on the accessibility of the single subsystems and an additional Lie-theoretic notion. In the second part of the talk, we generalize the previous ideas to infinitely many bilinear systems. This leads to infinite dimensional bilinear control systems of a particular structure. We will present first results for countably many parallel connected systems modeled on an appropriate Banach Lie group. The continuous case is still open.

Dor'on Adam. *Cuntz-Krieger dilations of Toeplitz-Cuntz-Krieger families via Choquet theory.*

Perhaps the simplest dilation result in operator theory is the dilation of an isometry to a unitary. However, when one generalizes an isometry to a Toeplitz-Cuntz-Krieger family of a directed graph, things become much more complicated.

The analogue of a unitary operator in this case is a (full) Cuntz-Krieger family, and a result of Skalski and Zacharias on C^*-correspondences supplies us with a such a dilation when the graph is row-finite and sourceless. We apply Arvesons non-commutative Choquet theory to answer this question for arbitrary graphs. We compute the non-commutative Choquet boundary of graph tensor algebras and are able to recover a result of Katsoulis and Kribs on the computation of the C-envelope of these algebras. This joint work with Guy Salomon.

Dym Harry. *Sampling Formulas in Vector Valued de Branges Spaces.*

A number of relatively recent papers have established connections between reproducing kernel Hilbert spaces of entire functions, de Branges spaces, sampling formulas and a class of symmetric operators with deficiency indices (1,1). This talk will discuss generalizations to reproducing kernel Hilbert spaces of entire vector valued functions and symmetric operators with deficiency indices (p,p).

The talk is based on an article with Santanu Sarkar.

Feintuch Abie. *Asymptotic Behavior of First and Second Order Spatially Invariant Systems.*

A spatially invariant linear system is a dynamical linear system whose state space is a linear (infinite) sequence space, usually a Banach space, and whose state operator commutes with the standard co-ordinate shift operator on the space. Such systems arise in a natural way in the study of serial pursuit and rendezvous problems, platoon model problems, infinite phonon models, as well as series connections of infinitely many linear electrical circuits. In this talk I will summarize my results on asymptotic behavior of some first and second order systems, much of which was done jointly with Bruce Francis, and list some open problems.

Fuhrmann Paul. *Thoughts on optimal control*

The purpose of this talk is to outline a, seemingly new, approach to a wide variety of optimal control problems for linear, causal, time-invariant systems. This approach has the advantages of not being restricted to finite-dimensional systems, and has extensions to optimization problems for various classes of transfer functions, including stable or antistable, general, positive real and bounded real functions. The technique used is based on translation semigroups and their Fourier-Plancherel transforms, the Paley-Wiener theorem, the Beurling-Lax characterization of invariant subspaces, the commutant lifting theorem, intertwining maps, their inversion using the corona theorem, unimodular embedding, their relation to Hankel operators, normalized corime factorizations over H^∞, as well as realizations based on model operators. In the rational case, one can derive state space formulas for all relevant maps based on solutions to various Riccati equations. The suggested approach is natural for clarifying the connection, for example by using duality considerations, between seemingly different aspects of system theory. A case in point is the relation between model reduction and robust control.

References

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Goldshtein Vladimir. *Conformal Geometry and Elliptic Operators.*

The talk is devoted to connections between hyperbolic geometry of bounded simply connected planar domains and spectral estimates for Laplace and p-Laplace operators in non-convex domains.

The classical results by L. E. Payne and H. F. Weinberger (1960) give the lower estimates of the first nontrivial eigenvalue of the Neumann Laplacian in convex domains in terms of its diameters. The Nikodim-type examples show that in the general case of simply connected planar domains the first non-trivial eigenvalue can not be estimated in terms of its diameters. We obtained estimates in terms of the conformal radii for a large class of bounded non convex domains with some additional restrictions on the conformal (hyperbolic) geometry that we call a conformal regularity. This class of domains includes quasicircles (images of the unit disc under quasiconformal homeomorphisms of the plane). For example, it means that boundaries of domains can have any Hausdorff dimension between 1 and 2. Two results will be discussed in details:1) Spectral stability of Dirichlet and Neumann boundary problems under additional conditions on the conformal geometry of domains. Lower estimates for the principal eigenvalue of p-Laplacian for the Neumann boundary problem under some regularity conditions. The machinery base on: Brennans conjecture, composition operators on Sobolev spaces L^1_p-weighted Poincar-Sobolev inequalities.

Joint work with Alexander Ukhlov

References: [1] V. I. Burenkov, V. Goldshtein, A. Ukhlov, Conformalspectral stability for the Dirichlet-Laplace operator, Mathematische Nachrichten 288 (2015), no. 16, 1822-1833. [2] V. Goldshtein, A. Ukhlov, Weighted Sobolev spacesand embedding theorems, Trans. Amer. Math. Soc., 361, (2009), 38293850. [3] V. Goldshtein, A. Ukhlov, Brennans conjecture forcomposition operators on Sobolev spaces. Eurasian Math. J. 3 (2012), no. 4, 3543. [4] V. Goldshtein, A. Ukhlov, On the First Eigenvalues of Free Vibrating Membranes in Conformal Regular Domains,Arch. Rational Mech. Anal. (2016), Volume 221, Issue 2, 893915.

Gombani Andrea. *A general approach to multivariable recursive interpolation.*

We consider here the problem of constructing a general recursive algorithm to interpolate a given set of data with a rational function. While many algorithms of this kind already exists, they are either providing non minimal-degree solutions (like the Schur algorithm), or exhibit jumps in the degree of the interpolants (or of the partial realization, as the problem is called when the interpolation is at infinity, see Rissanen and Gragg-Lindquist). By imbedding the solution into a larger set of interpolants, we show that the increase in the degree of this representation always equals the increase in the length of the data. We provide an algorithm to interpolate multivariable tangential sets of data with arbitrary nodes, generalising in a fundamental manner the results of Kuijper.

Grüne Lars. *Controlling stochastic ensembles via model predictive control for the Fokker-Planck-equation.*

For the control of ensembles governed by controlled stochastic differential equations we follow the approach to control the corresponding probability density function. To this end, we propose to use Model Predictive Control (MPC) for the Fokker-Planck equation. In this talk we start by describing the basic setup and illustrating the approach by numerical examples. Then, we provide results on the analysis of the stability and performance of the MPC approach.

Finally, we discuss the structure of the controller resulting from the MPC approach, particularly its dependence on space, time and on the probability density function of the ensemble under consideration.

The talk is based on joint work with Arthur Fleig

Hinrichsen Diederich. *A Gershgorin approach to networks of linear systems*

In this paper we develop a Gershgorin approach to networks of time-invariant linear systems with uncertain parameters. The approach can be extended to networks of *time-varying* linear systems and to networks of time-invariant *infinite dimensional* linear systems. In this talk I will not deal with the technicalities of recent research, but outline the basic approach and present some typical results. The talk is based on past joint work with A. J. Pritchard (University of Warwick) (†) and M. Karow (Technical University Berlin).

As an introduction we recall the classical inclusion theorems from linear algebra, due to S.A. Gershgorin (1931), A. Brauer (1947) and R.A. Brualdi (1982). These theorems deal with single matrices and specify easily computable regions in the complex plane that are guaranteed to include the eigenvalues of the matrix. Our aim is to extend these results to the spectral analysis of networks of linear systems with (possibly) uncertain parameters. Combining the Gershgorin approach with the ideas of *spectral value sets* and *stability radii* we obtain a general theory which provides computable formulas for networks of time-invariant linear systems with uncertain time-invariant or time-varying couplings. The classical inclusion theorems are rather special cases of these formulas. Additionally, they imply that the results of A. Brauer (1947) and R.A. Brualdi (1982) are sharp, i.e. under the hypotheses of Brauer or Brualdi their estimates cannot be improved.

Lewkowicz Izchak. *Dissipative Systems - Convex Invertible Cones point of view.*

Convex cones over a real unital algebra, which in addition are closed under inversion, may seem peculiar. However, Convex Invertible Cones (CICs) naturally appear in stability analysis of continuous-time physical systems.

With this motivation, in this talk we explore examples of CICs over some algebras and establish interconnections among them.

This indicates at the importance of the study of rational functions, of non-commuting variables, with certain positivity properties.

Lieb Julia. *Probability estimates for networks of linear systems over finite fields and applications to convolutional codes.*

Some properties such as reachability or observability are of considerable interest for dealing with linear systems. If one focuses on systems defined over finite fields, it becomes possible to count the number of systems with a certain property and therefore, one could estimate probabilities. Using a criterion for the reachability and observability of interconnected systems, we extend these probability estimations to networks of systems, such as parallel or series connection. Since there is a correspondence between linear systems and convolutional codes, these considerations could be transferred to interconnected convolutional codes. Hereby, the reachability and observability of the system correlate with the minimality and non-catastrophicity of the code.

Markiewicz Daniel. *Classification of C*-envelopes of tensor algebras arising from stochastic matrices.*

In this talk we discuss the C*-envelope of the (non-self-adjoint) tensor algebra associated via subproduct systems to a finite irreducible stochastic matrix P.

We showed previously that there are examples of such C*-envelopes that are not *-isomorphic to either the Toeplitz algebra or the Cuntz-Pimsner algebra, which was somewhat unexpected. In this talk we provide a detailed identification of the boundary representations of the tensor algebra inside the Toeplitz algebra, also known as its non-commutative Choquet boundary. We apply this characterization to clarify matters by describing the various C*-envelopes that can land between the Toeplitz and the Cuntz-Pimsner algebras. More precisely, we classify the C*-envelopes of tensor algebras up to *-isomorphism and stable isomorphism, in terms of the underlying matrices.

This talk is based on the paper: A. Dor-On and D. Markiewicz, "C*-envelopes of tensor algebras arising from stochastic matrices", arXiv:1605.03543 [math.OA], to appear in *Integral Equations and Operator Theory.*

Markus Alexander. *On the Norm of Linear Combinations of Projections and Some Characterizations of Hilbert Spaces.*

Let $\mathcal{B}$ be a Banach space and let P, Q $(P, Q \neq 0)$ be two complementary projections in $\mathcal{B}$ (i.e. $P + Q = I$). For $\dim \mathcal{B} > 2$ we show that formulas of the kind $\|aP + bQ\| = f(a, b, \|P\|)$ hold if and only if the norm in $\mathcal{B}$ can be induced by an inner product. We also consider the two-dimensional case.

This is a joint work with N.Krupnik.

Rosenthal Joachim. *Algebraic Systems Theory and Coding Theory.*

It is well known that a convolutional code is essentially a linear system defined over a finite field. Despite this well known connection convolutional codes have been studied in the past mainly by graph theoretic methods and in contrast to the situation of block codes there exist only few algebraic constructions. It is a fundamental problem in coding theory to construct convolutional codes with a designed distance.

In a first part of the talk we will explain the connection between convolutional codes and linear systems via a natural duality. In doing so convolutional codes can be viewed as submodules of R^n where $R := F[z]$ is a polynomial ring. The set of all convolutional codes of a fixed degree is parameterized by the Grothendieck Quot Scheme. If the degree is zero this scheme describes a

Grassmann variety. In this last situation the set of linear block codes of a fixed rate are parameterized.

In a second part of the talk we will cover applications of convolutional codes over large alphabets. E.g. such codes can be used for fault tolerant systems or for channels where errors appear as erasures. This has then a direct application for video streaming over packet switched systems like the Internet.

Shalit Orr. *Spaces of Dirichlet series with the complete Pick property.*

We consider reproducing kernel Hilbert spaces of Dirichlet series with kernels of the form $k(s,u) = \sum a_n n^{-s-\bar{u}}$, and characterize when such a space is a complete Pick space. We then discuss what it means for two reproducing kernel Hilbert spaces to be "the same", and introduce a notion of weak isomorphism.

Many of the spaces we consider turn out to be weakly isomorphic as reproducing kernel Hilbert spaces to the Drury-Arveson space H^2_d in d variables, where d can be any number in $\{1, 2, \dots, \infty\}$, and in particular their multiplier algebras are unitarily equivalent to the multiplier algebra of H^2_d. Thus, a family of multiplier algebras of Dirichlet series are exhibited with the property that every complete Pick algebra is a quotient of each member of this family. Finally, we determine precisely when such a space of Dirichlet series is weakly isomorphic as a reproducing kernel Hilbert space to H^2_d and when its multiplier algebra is isometrically isomorphic to the multiplier algebra of H^2_d.

This talk is based on a joint work with John McCarthy, to appear in Israel J Math.

Shamovich Eli. *Dilation of Semigroups of Contractions and System Theory*

In this talk we will discuss overdetermined conservative systems associated to a [image: d]-tuple of commuting operators on a Hilbert space, their associated compatibility systems and ways of solving them. We will then restrict ourselves to the case when the [image: d]-tuple consists of dissipative operators and show how under certain conditions one can construct a Hilbert space from the solutions of our system that will serve as a dilation space for the multiparameter semigroup of contractions generated by our [image: d]-tuple.

Smith Malcolm. *A new look at the Ladenheim catalogue: tribute to U. Helmke and R. Kalman.*

The talk will present a tutorial introduction to the Ladenheim catalogue (i.e. the set of 5-element electrical networks with at most two reactive elements and at most three resistors) which was at the centre of Kalman's research interests during his final decade and was the focus of a series of workshops initiated by Uwe Helmke starting in 2010. The talk will discuss some earlier work by Jiang and Smith on the catalogue, including the notion of a regular positive real function, and some more recent work with A. Morelli. The talk concludes with a discussion on the status of some outstanding conjectures of Kalman in connection with the catalogue.

Solel Baruch. *Matrix convex sets, Dilations and CP maps.*

We use matrix convex sets $S = \cup_n S_n \subset \cup_n M_n(C)$ (as in Wittstock and Effros-Winkler) to study

(1) the existence of interpolating unital completely positive maps mapping one given tuple of operatorsto another
and
(2) the existence of certain dilations of a given tuple of operators to a normal commuting tuple. Some of these results follow from understanding the dependence of S on S_1.

We were inspired by the recent work of Helton, Klep, McCullough and Schweighofer on free spectrahedra and an older work of Arveson on matrix ranges.

This is a joint work with A. Dor-On, K. Davidson and O. Shalit.

Staffans Olof. *State/Signal Invariant Properties of Input/State/Output Systems.*

In a linear continuous time i/s/o (input/stat/output) system $\Sigma_{i/s/o}$ the time derivative $\dot{x}(t)$ at time t of the state $x(t)$ is a linear function of the state $x(t)$ and the input $u(t)$, and the output $y(t)$ is a linear function of the state $x(t)$ and the input $u(t)$. From such a system one gets a s/s (state/signal) system $\Sigma_{s/s}$ by combining the input and output signals $u(t)$ and $y(t)$ into one interaction signal $w(t)$ and ingnoring the distinction between the "input" and the "output". Thus, to every i/s/o system $\Sigma_{i/s/o}$ there corresponds a unique s/s system $\Sigma_{s/s}$, but conversely, from every s/s system $\Sigma_{s/s}$ it is possible to construct many i/s/o systems $\Sigma_{i/s/o}$, which differ from each other by the way in which the interaction signal has been split into an input and an output. Each such i/s/o system $\Sigma_{i/s/o}$ is called an i/s/o representation of $\Sigma_{s/s}$.

By a "s/s invariant property" of an i/s/o system $\Sigma_{i/s/o}$ we mean a property of the following type: Let $\Sigma_{s/s}$ be the s/s system induced by $\Sigma_{i/s/o}$. We say that a certain property P of $\Sigma_{i/s/o}$ is a *s/s invariant property* of $\Sigma_{i/s/o}$ if not only $\Sigma_{i/s/o}$ itself has this property, but *every i/s/o representation of* $\Sigma s/s$ *has the same property.* Typical examples of s/s invariant properties are controllability, observability and minimality. Some other s/s invariant properties describe the existence and uniqueness of classical and generalized trajectories of the system.

Another class of properties of an i/s/o system $\Sigma_{i/s/o}$ is the following: We say, for example, that the i/s/o system $\Sigma_{i/s/o}$ is *well-posed in the s/s sense* if the corresponding s/s system $\Sigma_{i/s/o}$ is well-posed. As a consequence of this definition, if $\Sigma_{s/s}$ has *one* well-posed i/s/o representation, then *every i/s/o representation of* $\Sigma_{s/s}$ *is well-posed in the s/s sense* (because all these i/s/o representations induce the same s/s system). For example, the s/s system induce by a passive impedance or transmission i/s/o system is a passive s/s system, and every passive s/s system has a well/posed passive i/s/o scattering representations. Consequently every passive impedance and transmission i/s/o system is well-posed in the s/s sense. This fact tells us something about the existence and uniqueness of classical and generalized trajectories of passive impedance and transmission i/s/o systems, in spite of the fact that the impedance or transmission system need not be well-posed. There are some other properties, in addition to well-posedness, which are of the same type, i.e., an i/s/o

Tannenbaum Allen. *Emerging Problems in Networks and Network Dynamics*

Today's technological world is increasingly dependent upon the reliability, robustness, quality of service and timeliness of networks including those of power distribution, financial, transportation, communication, and social. For the time-critical functionality in transferring resources and information, a key requirement is the ability to adapt and reconfigure in response to structural and dynamic changes, while avoiding disruption of service and catastrophic failures. In this talk, we will outline some of the major problems for the development of the necessary theory and tools that will permit the understanding and managing of network dynamics in a multiscale manner.

Many interesting networks consist of a finite but very large number of nodes or agents that interact with each other. The main challenge when dealing with such networks is to understand and regulate the collective behavior. Our goal is to develop mathematical models and optimizational tools for treating the *Big Data* nature of large scale networks while providing the means to understand and regulate the collective behavior and the dynamical interactions (short and long-range) across such networks.

Accordingly, a central theme of the talk will be based on the analysis of interacting systems of particles or agents, in a dynamic environment, under a variety of models of their mutual interactions and selective environmental pressures. The theory and techniques will aim at capturing the essential collective behavior at various scales, and will address and study potential sources of perceived collective responses. Examples will be given from biology, finance, and transportation.

Tkachenko Vadim. *Transformation operators in inverse problems of high-order ODE.*

The main aim in the theory of inverse problems for ordinary differential operators is to find 1-to-1 correspondence between coefficients of a given such operator on one side, and the spectral data supplied by boundary problems generated by it together with an adequate boundary conditions, on another side.

Back in 1929 V.Ambartsumya made the first step in this theory proving the uniqueness theorem for problems generated by the well-known Sturm-Liouville operators $-d^2/dx^2 + q(x), x \in [0, a]$, with continuous function $q(x)$.

The fundamental results for the general Sturm-Liouville-Schrödinger operators were obtained by G.Borg, I.Gel'fand, B.Levitan, V.Marchenko, M.Krein in 40-50-ies years of the last century. Efficient techniques they developed to recover coefficients of the latter problems and of their second-order generalizations led to the blossoming of the theory itself and of its numerous applications.

The picture is different for operators of order $n > 2$. The main obstacle to the advancement here was discovered by V.Matsaev who proved that, generally speaking, the main technical tool used for $n = 2$ (the Volterra-type transformation operator) does not exist for $n > 2$.

The essential achievement in a study of inverse problems for general operators of high order belongs to Z.Leibenzon. His papers [1] - [2] containing a nonstandard system of boundary problems associated with such operators and a new notion of the transformation operator gave an effective method of **reconstructing** already existing operator and boundary conditions using the corresponding spectral data.

In the same context, the problem of **constructing** both operator and boundary conditions with the prescribed spectral data remained open.

Following ideas and results due to Z.Leibenzon we will present a solution of such inverse problems for selfadjoined operators of order 4 in the framework of Hilbert spaces.

The similar results are valid for arbitrary values $n > 4$.

[1] Z.L. Leibenzon, *Uniqueness of solutions of inverse problems of ordinary differential operators of order $n \geq 2$ and the transforms of such operators*, Dokl.Acad.Nauk SSSR, 142 (1962), 100-103.

[2] Z.L. Leibenzon, *An inverse problem of spectral analysis of ordinary differential operators of higher order*, Trans. Mosc. Math. Soc., 78-163 (1966).

Trentelman Harry. *Model Reduction of Linear Multi-Agent Systems by Clustering and Associated Error Bounds.*

In this talk, we discuss a model reduction technique for leader-follower networked multi-agent systems defined on weighted, undirected graphs with arbitrary linear multivariable agent dynamics. In the network graph of this network, nodes represent the agents and edges represent communication links between the agents. Only the leaders in the network receive an external input, the followers only exchange information with their neighbors. The reduced network is obtained by partitioning the set of nodes into disjoint sets, called clusters, and associating with each cluster a single, new, node in a reduced network graph. The resulting reduced network has a weighted, symmetric, directed network graph, and inherits some of the structure of the original network. We establish a priori upper bounds on the H-2 and H-infinity model reduction error for the special case that the graph partition is almost equitable. These upper bounds depend on the Laplacian eigenvalues of the original and reduced network, an auxiliary system associated with the agent dynamics, and the number of nodes that belong to the same clusters as the leaders in the network. Finally, we consider the problem of obtaining a priori upper bounds if we cluster using arbitrary, possibly non almost equitable, partitions.

Trumpf Jochen. *A general necessary and sufficient condition for controllability of networks of linear systems.*

In a 2007 paper, Hara et al. showed that a statically and linearly coupled homogeneous network of (identical) single-input single-output linear time-invariant agents is controllable if and only if the network input matrix has full row rank (meaning there are as many independent external inputs as there are agents); or if the agents are controllable and observable, and the network interconnection matrix and the network input matrix form a controllable pair. In their recent monograph on he mathematics of networks of linear systems, Fuhrmann and Helmke provide a necessary and sufficient condition for the controllability of networks of controllable and observable but otherwise general, heterogeneous linear time-invariant agents. The condition comes in the form of a generalised Hautus test, which we call the Fuhrmann- Helmke test.

It is easily seen that controllability of each agent is necessary for network controllability (in general, including for non-linear agents and network couplings), so this raises the question whether observability of the agents is also necessary (in the case of a rank-deficient network input matrix). We provide a general necessary and sufficient condition for the controllability of statically and linearly coupled networks of linear time-invariant agents in the form of a generalised Fuhrmann-Helmke test together with a relatively short proof based on behavioral theory and kernel representations. We show that observability of agents is necessary in the homogeneous case but not necessary in general. Likewise, controllability of the network matrix pair is not necessary beyond the single-input single-output homogeneous case.

Verriest Erik. *Partial state reachability and observability of cascades of linear systems.*

We characterize reachability and observability for a proper subset of component systems in the cascade connection (series and parallel connections) of multivariable linear discretetime systems. While such a characterization is readily given in state space form, it is hard to work with. Using tools from algebraic systems theory, partial state reachability and observability of a series connection are characterized in terms of more easily manageable conditions on Toeplitz operators and coprime factorizations of the component transfer functions. Our results extend earlier results by Callier and Nahum on reachability of the series connection of two systems, as well as a more recent characterization of the full observability on the series connection of linear systems. In addition, duality is briefly explored, showing that the notions of partial reachability and observability are not dual, unless all systems in the series connection are considered. We also present an extension of the classical PBH type tests.

Vinnikov Victor. *Interpolation and Realization Theorems for Noncommutative Functions*

Realization of a function as the transfer function of an input/state/output linear system is closely related to interpolation problems. More specifically, there is a close link between conservative (or unitary) realizations of contractive (or Schur class) functions and classical interpolation problems such as those of Nevanlinna–Pick and Caratheodory–Fejer. Following an extensive development of these ideas in the one-dimensional setting of the unit disc, they were generalized to multidimensional situations such as the unit ball or the unit polydisc in $\mathbb{C}^d$ following the seminal work of Agler. I will discuss a further generalization in the setting of free noncommutative function theory. A function of d free noncommuting variables is a function whose domain is contained in the set of d-tuples of square matrices of all sizes and whose values are square matrices of the same size, and that respects direct sums and simultaneous similarities. More generally, a noncommutative function is a function whose domain is contained in the set of square matrices over a vector space $\mathcal{V}$ and whose values are square matrices of the same size over a vector space $\mathcal{W}$, and that respects direct sums and similarities. Such functions admit a rich analytic theory that originated in the pioneering work of J.L. Taylor on noncommutative spectral theory, and that was actively developed in recent years. I will present a realization and interpolation theorem for contractive noncommutative functions on domains that are defined as "balls" with respect to some given noncommutative function. In the case when the space $\mathcal{V}$ is finite dimensional and the defining function for the domain is matrix-valued (rather than operator-valued) and linear or polynomial, we recover the previous results of Ball–Groenewald–Malakorn and of Agler–McCarthy. A consequence of our general theorem is an amazing extension property for bounded noncommutative functions on matrix convex domains which is reminiscent yet strikingly different from Cartan's theorem in several complex variables.

This is a joint work with J. Ball and G. Marx (Virginia Tech).

Weiss George. *Systems described by very rapid switching between two m-dissipative operators.*

In this talk we consider the system with state trajectory $x(t)$ that evolves in a Hilbert space X and satisfies $x'(t) = Ax(t)$ half of the time, and $x'(t) = Bx(t)$ half of the time. Here A and B are m-dissipative operators (i.e., generators of contraction semigroups) and the meaning of "half of the time" can be that we have very fast switching between the two dynamics, with equal amount of time spent in the two dynamics, and the switching frequency tends to infinity. We shall also explore other possible meanings of "half of the time". Intuitively, we expect the limit system to be described by $x'(t) = (A + B)/2x(t)$. But it is very difficult to define the operator $(A + B)/2$. This is related to the famous theorem of P.R. Chernoff from 1968, which generalizes the Trotter formula. We also discuss extensions to switching between two systems, with inputs and outputs.

Zhou Huacheng. *Proportional control based output tracking and disturbance rejection for 1-D anti-stable wave equation.*

In this talk, we will solve the output tracking and disturbance rejection problem for a system (plant) described by a 1-D anti-stable wave equation, with reference and disturbance signals that belong to $W^{1,\infty}(0,\infty)$ and $L^{\infty}(0,\infty)$, respectively. We explore an approach based on proportional control. It is shown that a proportional gain controller can achieve exponentially the output tracking while rejecting the disturbance. As a byproduct, we obtain a new output feedback stabilizing control law by which the resulting closed-loop system is exponentially stable using only two displacement output signals. Numerical experiments are carried out to illustrate effectiveness of the proposed control law.