Homotopy normal maps

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Abstract

A group property made homotopical is a property of the corresponding classifying space. This train of thought leads to the definition of normal maps between discrete groups $N \to G$, as was introduced by Farjoun and Segev, being those maps for which $BN \to BG$ is the inclusion of the homotopy kernel (i.e., homotopy fiber) of some map $BG \to W$ with $W$ a connected space. Normal maps give a homotopical analogue to the inclusion of a normal subgroup and in particular induce a compatible topological group structure on the homotopy quotient $G//N \equiv EN \times_N G$.

In this talk we deal with a continuous generalization of the above, called homotopy normal maps, which are topological group maps (or loop maps) $N \to G$ being normal in the same sense. We characterize these maps by a compatible simplicial loop space structure on $Bar_\bullet(N, G)$, invariant under homotopy monoidal functors $Top \to Top$, i.e., functors which preserve finite products up to homotopy (e.g., localizations and completions). In the course of characterizing normality, we define a notion of homotopy action of a loop space on a space phrased in terms of Segal’s 1-fold delooping machine. Homotopy actions are ‘flexible’ in the sense they are invariant under homotopy monoidal functors but can also rigidify to (strict) group actions.