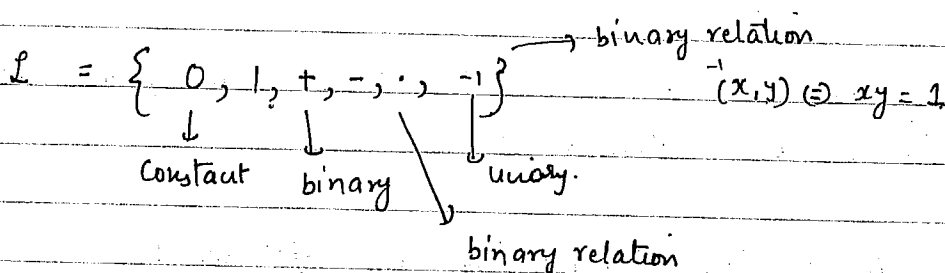
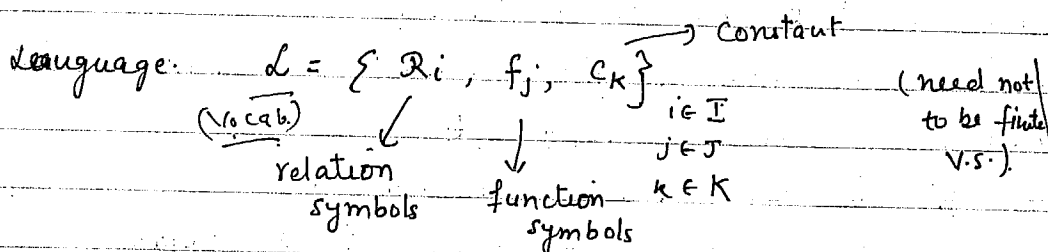


"Model Theory" (of valued fields) 09/06/09.



"functions should be total"

We will always assume that $=$ is part of language

Given a language L , a structure for L is

- (i) A nonempty set M (we always assume infinite)
- (ii) (a) For every constant symbol $c_k \in L$ an element $a \in M$ (which we call the interpretation of c_k in M)

(b) For any function symbol $f_j \in L$, if f_j is n -ary, a total function $f: M^n \rightarrow M$

(c) For every relation $R_i \in L$, if R is m -ary a total function $f: M^m \rightarrow \{0, 1\}$ equivalent to - for every such R_i we observe a subset of M^m .

Def of truth in a structure \mathcal{B} -

Formula :- Def of a formula is inductive

need to define terms

(i) Every constant symbol is "term"

(ii) Any variable from a given (usually explicitly) list of variables is a term x, y, z, t

(iii) If f_i is an n -ary function in L and t_1, t_2, \dots, t_n are terms then $f_i(t_1, t_2, \dots, t_n)$ is a term

exple: - If L is the language of rings, all polynomials over \mathbb{Z} are in L and that's it.

A formula is defined inductively as follows:

1) If R_i is an m -ary relation in L and t_1, t_2, \dots, t_m are terms in L then

$R_i(t_1, t_2, \dots, t_m)$ is an atomic formula (need truth value)

2) If ϕ_1, ϕ_2 are formulas then so are $\neg \phi_1$ and $\{\phi_1 \wedge \phi_2\}$ where $\{\neg, \wedge\}$ are the logical connectives (in the language)

If ϕ_1 is a formula then $(\exists x) \phi_1$ and $(\forall x) \phi_1$ are formulas.

a formula ϕ we have to define what are free variables

of ϕ (or when is a variable free in ϕ).

(x appears in ϕ) is not in the range of any quantifier $(\exists x)$ or $(\forall x)$

$$\phi(t)$$
$$(\phi(y) \wedge (x=x))$$

$$\exists n \rightarrow \forall (u) (u = u) \wedge (x < y)$$

↑
free variable

on the language of ring

$$\exists x_1, \dots, x_m \exists y (x_m^2 + \dots + x_1 x = 0)$$

x - free variable

Defⁿ:- A formula with no free variables is called statement Φ

Let \mathcal{M} be a structure for a voc \mathcal{L} and $\phi(x_1, x_2, \dots, x_k)$ a formula with free variables x_1, x_2, \dots, x_k . A substitution is a function from the set of variables to \mathcal{M} and we have to define inductively $s(\phi(x_1, x_2, \dots, x_k))$

(i) If t is a term x_i so we have $s(x_i) \in \mathcal{M}$

(ii) If t is a constant symbol c_k $s(c_k) = c_k(\mathcal{M})$

(iii) If $\phi = f_i(t_1, \dots, t_m)$ $s(\phi) = f_i^{\mathcal{M}}(s(t_1), \dots, s(t_m))$

$\rightarrow \exists x (x^2 = 7) \wedge (x + y = z)$

substitution in variables not in range of quantifiers.

Definition of truth value:- If R_i is an n -ary relation, t_1, t_2, \dots, t_m terms and s is a substitution then $\mathcal{M} \models s(R_i(t_1, \dots, t_m))$ is true (written $\mathcal{M} \models s(R_i(t_1, \dots, t_m))$) if $(s(t_1), \dots, s(t_m)) \in R_i^{\mathcal{M}}$ (or $R(s(t_1), \dots, s(t_m)) = 1$).

\uparrow
the integers of R_i in \mathcal{M} .

If ϕ_1, ϕ_2 are formulas then $\mathcal{M} \models s(\phi_1 \wedge \phi_2)$ if $\mathcal{M} \models s(\phi_1)$ and $\mathcal{M} \models s(\phi_2)$

$\mathcal{M} \models s(\neg \phi)$ if $\mathcal{M} \not\models s(\phi)$

$\mathcal{M} \models s(\exists x \phi(x, y_1, \dots, y_n))$ if there exists a substitution s' such that $s'(y_i) = s(y_i)$ for all $1 \leq i \leq n$

and $\mathcal{M} \models s'(\phi(x, y_1, \dots, y_n))$

$\mathcal{M} \models s(\forall x \phi(x, y_1, \dots, y_n))$ for any s' as above.

substitution - elements of \mathcal{M} .

For $L = \{ \} < J$ it is impossible to say well ordered.